Dimensions, units and checking a formula

85 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-dimensions/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Use $F = ma$ to write one newton in the base SI units kg, m and s.

Answer .....

A2 Needed today [2 marks]

Simplify $(\mathrm{L}^2\mathrm{T}^{-1})^3 \times \mathrm{L}^{-4}\mathrm{T}^2$.

Answer .....

A3 From A-Level Maths [2 marks]

A car's speed rises steadily from $3\ \mathrm{m\,s^{-1}}$ to $11\ \mathrm{m\,s^{-1}}$ in 4 seconds. Find its acceleration and the distance it travels.

Answer .....

A4 Needed today [1 mark]

Density is mass divided by volume. Write the SI units of density.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Dimensions of a quantity

EXAMPLE 1 · I DO

Find the dimensions of momentum, mv, and of the work done by a force, Fs.

Hint Start from the base dimensions: mass M, length L, time T. Velocity is length over time, and a force is mass times acceleration. Write the answer as MLT^{-1}, with no square brackets round the letters.

YOUR TURN · WE DO

B1 Find the dimensions of (i) power, the work done per unit time (ii) pressure, the force per unit area. [4 marks]

Answer .....

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C1 [4 marks]

(i) The coefficient of friction is $\mu = \dfrac{F}{R}$, where F is the friction and R the normal reaction. Show that μ is dimensionless. (ii) A wheel turns through an angle $\theta = \omega t$ in time t. An angle is an arc length divided by a radius. Find the dimensions of ω.

Answer .....

C2 [4 marks]

The weight of a body of mass m is mg, and g is often given in $\mathrm{N\,kg^{-1}}$. By writing the newton in base units, find the dimensions of g. Hence show that mgh has the dimensions of energy.

Answer .....

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C3 Sam writes: “Density is measured in $\mathrm{kg\,m^{-3}}$, so $[\rho] = \mathrm{M} \times \mathrm{M^{-3}} = \mathrm{M^{-2}}$.” Explain Sam's mistake and give the correct dimensions of density. Hence show that ρv^2, where v is a speed, has the same dimensions as pressure. [5 marks]

Answer .....

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PART 2 Dimensions and SI units of a constant

EXAMPLE 2 · I DO

Two particles of masses m_1 and m_2 a distance d apart attract each other with a force $F = \dfrac{Gm_1m_2}{d^2}$. Find the dimensions of G.

Hint G is a named constant, so it can have dimensions. Make G the subject, then put in the dimensions of everything else.

YOUR TURN · WE DO

B2 A particle of mass m on a spring of stiffness s moves to and fro with period $t = 2\pi\sqrt{\dfrac{m}{s}}$. Find the dimensions of s. [4 marks]

Answer .....

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EXAMPLE 3 · I DO

Use $[G] = \mathrm{M}^{-1}\mathrm{L}^{3}\mathrm{T}^{-2}$ to write the SI units of G. Give them also in terms of newtons.

Hint Each dimension has a base unit: M is kg, L is m, T is s. Keep the powers.

YOUR TURN · WE DO

B3 A car moving at speed v meets air resistance $R = kv^2$. Find the dimensions of k, and its SI units. [4 marks]

Answer .....

PRACTICE · YOU DO

C4 The air resistance on a small ball of radius r moving at speed v is modelled as $R = cr^2v^2$, where c is a constant. Find the dimensions of c, and its SI units. [4 marks]

Answer .....

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C5 The resistance to a boat moving at speed v is modelled as $R = a + bv + cv^2$, where a, b and c are constants. Find the dimensions of a, b and c, and give the SI units of c. [5 marks]

Answer .....

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PART 3 Checking a formula

EXAMPLE 4 · I DO

Show that the formula $P = Fv$, for the power of a force F moving at speed v, is dimensionally consistent.

Hint Find the dimensions of the left side and of the right side separately, then compare.

YOUR TURN · WE DO

B4 Show that $s = ut + \frac{1}{2}at^2$ is dimensionally consistent. Explain why the check says nothing about the $\frac{1}{2}$. [4 marks]

Answer .....

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EXAMPLE 5 · I DO

A student writes the energy of a body of mass m at height h as $E = mgh + gh$. Show that this cannot be correct.

Hint Terms that are added must have the same dimensions. Find the dimensions of each term and say which one is different.

YOUR TURN · WE DO

B5 A ball is thrown upwards at speed u. A student claims it returns to the thrower after time $t = \frac{2u}{g} + \frac{u^2}{g}$. Show that this cannot be correct, saying which term is wrong. [4 marks]

Answer .....

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C6 Alex writes: "$v^2 = u^2 + 2as$ is dimensionally consistent, so it must be the correct formula." Show that $v^2 = u^2 + 4as$ is also dimensionally consistent, and explain why Alex's reasoning is wrong. [4 marks]

Answer .....

C7 A satellite moves in a circle of radius r about a planet of mass M. Its period is $t = 2\pi\sqrt{\dfrac{r^3}{GM}}$, where $[G] = \mathrm{M}^{-1}\mathrm{L}^3\mathrm{T}^{-2}$. Show that this formula is dimensionally consistent. [5 marks]

Answer .....

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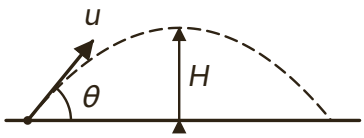
D1 A particle is projected from horizontal ground with speed u at an angle θ above the horizontal. It reaches a greatest height H.

(a) Explain why θ and $\sin\theta$ are dimensionless. [2 marks]



(b) Show that $H = \dfrac{u^2 \sin^2\theta}{2g}$ is dimensionally consistent. [3 marks]

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(c) A student suggests instead $H = \dfrac{u\sin\theta}{2g} + \dfrac{mu^2}{2g}$, where m is the mass of the particle. Show that this cannot be correct. [3 marks]

(d) Explain why dimensional analysis cannot confirm the 2 in the formula in part (b). [2 marks]

Total for D1: 10 marks

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D2

> Alex writes: “Every term in this equation is an energy, so it is dimensionally consistent.”

A body of mass m is pushed by a force F through a distance s while it rises a height h, and it ends with speed v. A student proposes the work equation

$$W = Fs + mgh + \tfrac{1}{2}v^2.$$

(a) Find the dimensions of Fs. [2 marks]

Answer .....

(b) Explain why Alex is wrong. Name the term that is not an energy. [3 marks]

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(c) The last term is replaced by $\frac{1}{2}kv^2$. Find the dimensions of k for the equation to be consistent, and suggest what k is. [3 marks]

Answer .....

Total for D2: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

A particle moves along a line so that its displacement at time t is

$$x = A\sin(\omega t) + Bt^2 + Ce^{-kt},$$

where A, B, C, ω and k are constants. Find the dimensions of each constant. The particle's velocity at $t = 0$ is $A\omega - kC$. Show that this expression has the dimensions of velocity.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Terms that are added all have the same dimensions as x.

Hint 2 You can only take the sine or the exponential of a number: ωt and kt must be dimensionless.

Hint 3 Find $[\omega]$ and $[k]$ first. Then $[A]$ and $[C]$ follow, because $\sin(\omega t)$ and e^{-kt} have no dimensions.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the dimensions of a quantity in terms of M, L and T from its definition or a formula.	○ ○ ○
I can write the SI units of a quantity from its dimensions.	○ ○ ○
I can check whether a formula is dimensionally consistent, term by term.	○ ○ ○
I can explain what a dimension check can show and what it cannot.	○ ○ ○

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