Kinetic and potential energy, and conservation of energy

87 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-energy/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the work done in lifting a $4\ \text{kg}$ box $3\ \text{m}$ vertically at constant speed.

Answer .....

A2 Last lesson [2 marks]

A $12\ \text{kg}$ box is pulled $5\ \text{m}$ up a slope inclined at $20°$. Find the work done against gravity.

Answer .....

A3 From A-Level Maths [2 marks]

A ball is dropped from rest and falls $20\ \text{m}$. Use suvat to find its speed.

Answer .....

A4 From A-Level Maths [2 marks]

Change 1.2 radians into degrees, and find $\cos 1.2$ (in radians).

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 Kinetic energy, potential energy and conservation

EXAMPLE 1 · I DO

A $1200\ \text{kg}$ car travels at $15\ \text{m s}^{-1}$. Find its kinetic energy. A $60\ \text{kg}$ climber rises $250\ \text{m}$. Find her gain in gravitational potential energy.

Hint KE $= \frac{1}{2}mv^2$ and GPE $= mgh$. Both are in joules.

YOUR TURN · WE DO

B1 (i) A $0.4\ \text{kg}$ ball speeds up from $3\ \text{m s}^{-1}$ to $8\ \text{m s}^{-1}$. Find its gain in kinetic energy. (ii) A $75\ \text{kg}$ diver drops $10\ \text{m}$. Find her loss of gravitational potential energy. [4 marks]

Answer .....

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EXAMPLE 2 · I DO

A $0.5\ \text{kg}$ ball is thrown from the top of a $20\ \text{m}$ cliff with speed $8\ \text{m s}^{-1}$. Find its speed when it reaches the sea below.

Hint KE at the start + PE lost = KE at the end. The direction of the throw does not matter, and the mass cancels.

YOUR TURN · WE DO

B2 A ball is thrown with speed $14\ \text{m s}^{-1}$ at $40°$ above the horizontal from a point $1.5\ \text{m}$ above the ground. Ignoring air resistance, find its speed when it is $5\ \text{m}$ above the ground. [4 marks]

Answer .....

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C1 A ball is thrown vertically upwards at $11\ \mathrm{m\,s^{-1}}$ from a point $1.2\ \mathrm{m}$ above the ground. Use energy to find (i) its greatest height above the ground, (ii) its speed when it hits the ground. [4 marks]

Answer .....

C2 Two particles are released from rest at the same height, $3\ \mathrm{m}$ above the ground, on two smooth slopes. One has mass $1\ \mathrm{kg}$ and slides down a slope at 20°. The other has mass $5\ \mathrm{kg}$ and slides down a slope at 50°. Show that they reach the ground with the same speed, and find that speed. [4 marks]

Answer .....

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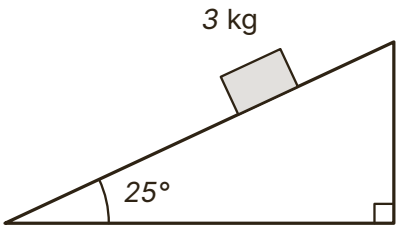
PART 2 Energy on slopes

EXAMPLE 3 · I DO

A $3\ \text{kg}$ block is released from rest and slides $4\ \text{m}$ down a smooth slope inclined at $25°$. Find its speed.



Hint The block drops $4\sin 25°$ metres. Use the vertical drop, not the distance along the slope.

YOUR TURN · WE DO

B3 A particle is projected up a line of greatest slope of a smooth plane inclined at $35°$, with speed $9\ \text{m s}^{-1}$. Find how far up the plane it travels before it stops. [4 marks]

Answer .....

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EXAMPLE 4 · I DO

A skier of mass $70\ \text{kg}$ starts from rest at A on a smooth slope. A is $30\ \text{m}$ above the bottom B. She passes C, which is $12\ \text{m}$ above B, then reaches B. Find her speed at C and at B.

Hint Take B as the zero level. Then the PE at A is $70g \times 30$ and at C it is $70g \times 12$.

YOUR TURN · WE DO

B4 A $2\ \text{kg}$ particle moves on a smooth track. It passes P, $4\ \text{m}$ above the ground, at $6\ \text{m s}^{-1}$. It then passes Q at ground level and R, $5\ \text{m}$ above the ground. Find its speed at Q and at R, and the greatest height above the ground it could reach on the track. [5 marks]

Answer .....

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C3 A bead slides on a smooth wire. At A its speed is $3\ \mathrm{m\,s^{-1}}$ and at B its speed is $7\ \mathrm{m\,s^{-1}}$. State whether B is above or below A, and find the difference in their heights. [4 marks]

Answer .....

C4 A bead is threaded on a smooth curved wire in a vertical plane. It passes A at $2\ \mathrm{m\,s^{-1}}$ and then B, which is $1.5\ \mathrm{m}$ lower than A. Find its speed at B. Explain why the suvat equations cannot be used to find it. [4 marks]

Answer .....

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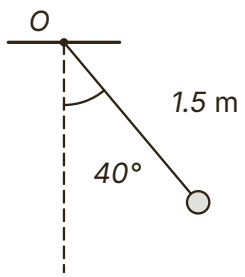
PART 3 Pendulums

EXAMPLE 5 · I DO

A pendulum has a light string of length $1.5\ \text{m}$. The bob is released from rest with the string taut at $40°$ to the downward vertical. Find its speed at the lowest point.



Hint The bob starts $1.5\cos 40°$ below O and ends 1.5 below O. So it drops $1.5(1-\cos 40°)$.

YOUR TURN · WE DO

B5 A pendulum has a light string of length $0.8\ \text{m}$ and a bob of mass $0.2\ \text{kg}$. It is released from rest with the string at 0.9 radians to the downward vertical. Find the speed of the bob when the string makes 0.3 radians with the downward vertical. [5 marks]

Answer .....

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C5 A pendulum has a string of length $2.5\ \text{m}$ and is released from rest at 30° to the downward vertical. Sam writes: [4 marks]

$$\text{height fallen} = 2.5\cos 30^\circ = 2.17\ \text{m},\ \text{so}\ v = \sqrt{2 \times 9.8 \times 2.17} = 6.51\ \text{m}\,\text{s}^{-1}.$$

Explain Sam's mistake, and find the speed of the bob at its lowest point.

Answer .....

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C6 A marble is released from rest on the inside surface of a smooth hemispherical bowl of radius $0.5\ \mathrm{m}$, fixed with its rim horizontal. Its speed at the bottom of the bowl is $2.8\ \mathrm{m\,s^{-1}}$. Find the angle between the downward vertical and the radius to the point where it was released. [5 marks]

Answer .....

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C7 A particle P hangs at rest from a fixed point O on a light inextensible string of length $0.9\ \mathrm{m}$. P is given a horizontal speed of $2.5\ \mathrm{m\,s^{-1}}$. The string stays taut. Find the angle between the string and the downward vertical when P first comes to rest. [6 marks]

Answer .....

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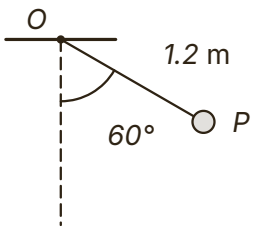
D1 A particle P of mass $0.25\ \text{kg}$ hangs from a fixed point O on a light inextensible string of length $1.2\ \text{m}$. P is pulled aside so that the string is taut at 60° to the downward vertical, and released from rest.

(a) Show that the speed of P at its lowest point is $\sqrt{1.2g}$. [3 marks]



(b) Find the speed of P when the string makes 25° with the downward vertical. [3 marks]

Answer .....

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(c) Hence find the angle between the string and the downward vertical when the speed of P is first $2\ \mathrm{m\,s^{-1}}$. [3 marks]

Answer .....

Total for D1: 9 marks

D2

> Alex writes: "The ball hits the ground fastest if Jo throws it straight down, because then all its speed is downwards."

Jo throws a ball from the top of a $25\ \mathrm{m}$ tower with speed $12\ \mathrm{m\,s^{-1}}$.

(a) Jo throws the ball horizontally. Find its speed when it hits the ground. [3 marks]

Answer .....

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(b) Explain why Alex is wrong. [2 marks]

(c) The ball has mass $0.15\ \text{kg}$. Find its kinetic energy when it hits the ground. [2 marks]

Answer .....

(d) State one modelling assumption you have made, and its effect on your answer to (a). [2 marks]

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A ball of mass $0.2\ \text{kg}$ is thrown from a point $2\ \text{m}$ above horizontal ground, with speed u at 55° above the horizontal. It passes over a wall $5\ \text{m}$ high with speed $9\ \text{m s}^{-1}$, and then lands on the ground. Ignoring air resistance, find u, the speed with which the ball lands, and the least kinetic energy of the ball during its flight. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Use energy from the point of throwing to the top of the wall. The ball rises $3\ \text{m}$ there.

Hint 2 Use energy again from the point of throwing to the ground. The direction of motion does not matter.

Hint 3 The horizontal part of the velocity never changes. Where is the vertical part zero?

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find kinetic energy and gravitational potential energy.	○ ○ ○
I can use conservation of energy when no resistance acts, in any direction of motion.	○ ○ ○
I can find the height of a pendulum as $r(1-\cos\theta)$, in degrees or radians.	○ ○ ○
I can name a zero level and use energy where suvat cannot be used.	○ ○ ○

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