Collisions with a wall and multiple collisions

97 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-multiple-collisions/

Date

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A Do Now

Quick questions to start.

A1 From chapter 3 [3 marks]

Smooth spheres A and B of equal radius have masses $3\ \text{kg}$ and $1\ \text{kg}$. A moves at $4\ \text{m s}^{-1}$ and hits B, which is at rest. The coefficient of restitution is $\frac{1}{2}$. Find the velocities of A and B after the collision.

Answer .....

A2 Needed today [2 marks]

Solve $6e^2 + e - 2 \leqslant 0$, given that $0 \leqslant e \leqslant 1$.

Answer .....

A3 From A-Level Maths [2 marks]

Two particles are $2.4\ \text{m}$ apart. They move towards each other along a straight line at $0.5\ \text{m s}^{-1}$ and $0.7\ \text{m s}^{-1}$. Find the time until they meet.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 A sphere hitting a wall

EXAMPLE 1 · I DO

A smooth sphere of mass $0.5\ \text{kg}$ moves at $8\ \text{m s}^{-1}$ on a smooth horizontal surface and hits a smooth vertical wall at right angles. The coefficient of restitution between the sphere and the wall is 0.6. Find the speed of the sphere after the impact, and the impulse that the sphere exerts on the wall.

Hint Take the direction away from the wall as positive. The sphere rebounds at e times its speed, so its velocity changes sign.

YOUR TURN · WE DO

B1 A smooth sphere of mass $0.4\ \text{kg}$ moving at $7.5\ \text{m s}^{-1}$ hits a smooth vertical wall at right angles and rebounds. The impulse on the sphere has magnitude $5.4\ \text{N s}$. Find the speed of the sphere after the impact and the coefficient of restitution between the sphere and the wall. State the impulse on the wall. [5 marks]

Answer .....

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C1 A smooth sphere of mass $0.2\ \text{kg}$ moving at $4\ \text{m s}^{-1}$ hits a smooth vertical wall at right angles. The coefficient of restitution is 0.75. Sam writes: "After the impact the velocity is $0.75 \times 4 = 3$, so the impulse on the sphere is $0.2 \times 3 - 0.2 \times 4 = -0.2\ \text{N s}$." Explain Sam's mistake, and find the impulse on the sphere and the impulse on the wall. [4 marks]

Answer .....

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C2 A smooth sphere moves on a smooth horizontal floor between two parallel smooth vertical walls X and Y, which are $3\ \mathrm{m}$ apart. It moves at right angles to both walls. It leaves wall X at $6\ \mathrm{m\,s^{-1}}$. The coefficient of restitution between the sphere and each wall is $\frac{1}{2}$. Treating the sphere as a particle, find the time from leaving X until it next hits X, and its speed after it rebounds from X. [4 marks]

Answer .....

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PART 2 Several collisions

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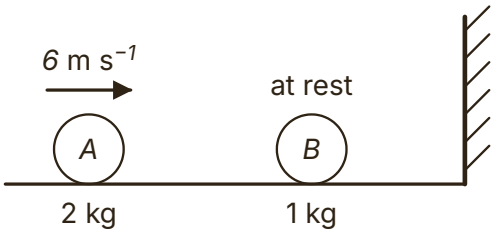
EXAMPLE 2 · I DO

Smooth spheres A and B of equal radius have masses $2\ \text{kg}$ and $1\ \text{kg}$. B is at rest between A and a smooth vertical wall that is perpendicular to the line of centres. A moves towards B at $6\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{2}$, and between B and the wall is $\frac{2}{3}$. Find the velocities of A and B after their second collision.



Hint Do one impact at a time, with a new before-and-after picture for each. Keep the direction towards the wall as positive all the way through.

YOUR TURN · WE DO

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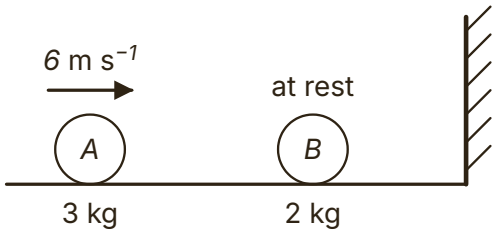
B2 [6 marks]

Smooth spheres A and B of equal radius have masses $3\ \text{kg}$ and $2\ \text{kg}$. B is at rest between A and a smooth vertical wall that is perpendicular to the line of centres. A moves towards B at $6\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{2}{3}$, and between B and the wall is also $\frac{2}{3}$. Find the velocities of A and B after their second collision.



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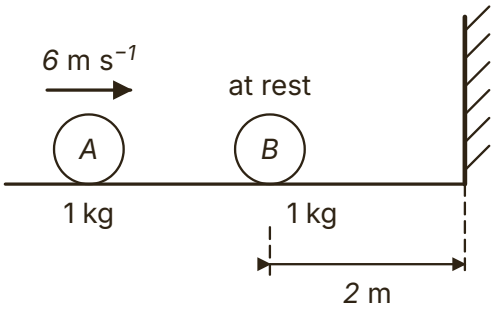
EXAMPLE 3 · I DO

Smooth spheres A and B of equal radius each have mass $1\ \text{kg}$. B is at rest at a distance of $2\ \text{m}$ from a smooth vertical wall that is perpendicular to the line of centres. A moves towards B at $6\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{3}$, and between B and the wall is $\frac{1}{2}$. Treating the spheres as particles, find how far from the wall A and B collide for the second time.



Hint Find both velocities after the first collision. Then find where A is at the moment B reaches the wall.

YOUR TURN · WE DO

Answer .....

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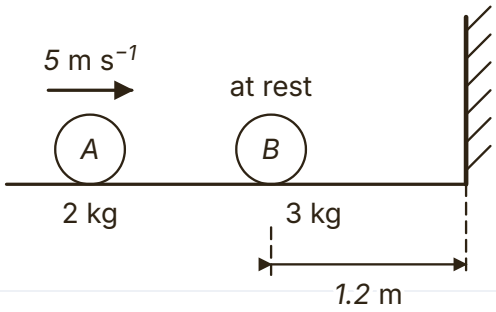
B3 [6 marks]

Smooth spheres A and B of equal radius have masses $2\ \text{kg}$ and $3\ \text{kg}$. B is at rest at a distance of $1.2\ \text{m}$ from a smooth vertical wall that is perpendicular to the line of centres. A moves towards B at $5\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{2}$, and between B and the wall is $\frac{2}{3}$. Treating the spheres as particles, find how far from the wall A and B collide for the second time.



PRACTICE · YOU DO

C3 [4 marks]

Three smooth spheres A, B and C of equal radius each have mass $0.5\ \text{kg}$. They lie at rest in a straight line, with B between A and C. A is projected towards B at $4\ \text{m s}^{-1}$. The coefficient of restitution between each pair is 1. Show that after two collisions A and B are at rest and C moves at $4\ \text{m s}^{-1}$, and state whether there are any further collisions.

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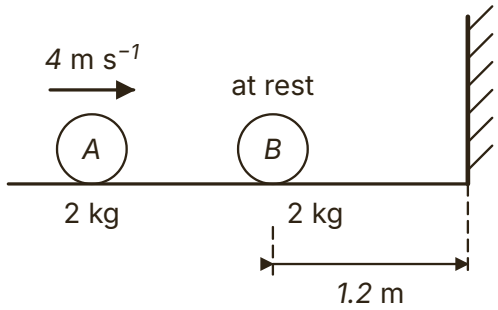
C4 Smooth spheres A and B of equal radius each have mass $2\ \text{kg}$. B is at rest at a distance of $1.2\ \text{m}$ from a smooth vertical wall that is perpendicular to the line of centres. A moves towards B at $4\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{2}$. Treat the spheres as particles.

Answer .....



(a) Find the velocities of A and B after they collide. [3 marks]

Answer .....

(b) Find the distance of A from the wall at the moment B hits the wall. [2 marks]

Answer .....

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(c) A and B collide for the second time at a distance of $0.3\ \text{m}$ from the wall. Find the coefficient of restitution between B and the wall. [4 marks]

Answer .....

Total for C4: 9 marks

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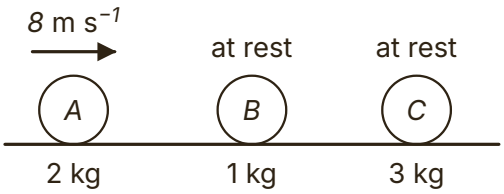
PART 3 Deciding whether there is another collision

EXAMPLE 4 · I DO

Three smooth spheres A, B and C of equal radius lie in a straight line on a smooth horizontal surface, with B between A and C. Their masses are $2\ \text{kg}$, $1\ \text{kg}$ and $3\ \text{kg}$. B and C are at rest and A moves towards B at $8\ \text{m s}^{-1}$. The coefficient of restitution between each pair of spheres is $\frac{1}{2}$. Determine whether A and B collide again.



Hint Find both velocities after the first collision, then both after B hits C. Then compare the velocities of A and B, in words.

YOUR TURN · WE DO

B4 Three smooth spheres A, B and C of equal radius lie in a straight line on a smooth horizontal surface, with B between A and C. Their masses are $1\ \text{kg}$, $3\ \text{kg}$ and $1\ \text{kg}$. B and C are at rest and A moves towards B at $8\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{2}$, and between B and C is $\frac{1}{3}$. Determine whether there are any further collisions. [5 marks]

Answer .....

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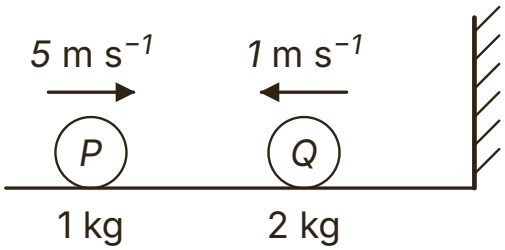
EXAMPLE 5 · I DO

Smooth spheres P and Q of equal radius have masses $1\ \text{kg}$ and $2\ \text{kg}$. They move towards each other along a line that is perpendicular to a smooth vertical wall, with Q between P and the wall. P moves at $5\ \text{m s}^{-1}$ and Q at $1\ \text{m s}^{-1}$. The coefficient of restitution between P and Q, and between Q and the wall, is e. Find the set of values of e for which P and Q collide again.



Hint Towards the wall is positive, so Q starts at -1. Find both velocities in terms of e. After the wall Q's velocity is $-e$ times its velocity before.

YOUR TURN · WE DO

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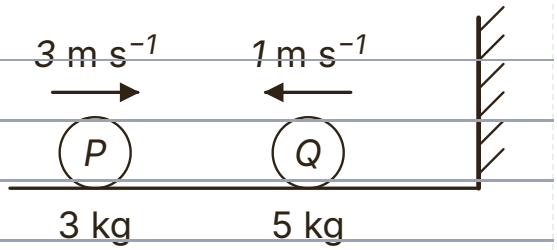
PRACTICE · YOU DO

C5 Smooth spheres A and B of equal radius have masses 1 kg and 3 kg. B is at rest between A and a smooth vertical wall, perpendicular to the line of centres. A moves towards B at 8 m s^{-1}. The coefficient of restitution between A and B is $\frac{1}{2}$, and between B and the wall is $\frac{1}{3}$. Sam writes: "B moves off the wall and comes back towards A, so A and B must collide again." Determine whether Sam is right. [5 marks]

B5 Smooth spheres P and Q of equal radius have masses 3 kg and 5 kg. They move towards each other along a line that is perpendicular to a smooth vertical wall, with Q between P and the wall. P moves at 3 m s^{-1} and Q at 1 m s^{-1}. The coefficient of restitution between P and Q, and between Q and the wall, is e. Find the set of values of e for which P and Q collide again. [6 marks]



Answer .....

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C6 Smooth spheres A and B of equal radius have masses $1\ \text{kg}$ and $2\ \text{kg}$. B is at rest between A and a smooth vertical wall that is perpendicular to the line of centres. A moves towards B at $6\ \text{m s}^{-1}$. The coefficient of restitution between A and B is e, and between B and the wall is $\frac{1}{4}$. Find the set of values of e for which A and B collide again. [5 marks]

Answer .....

Answer .....

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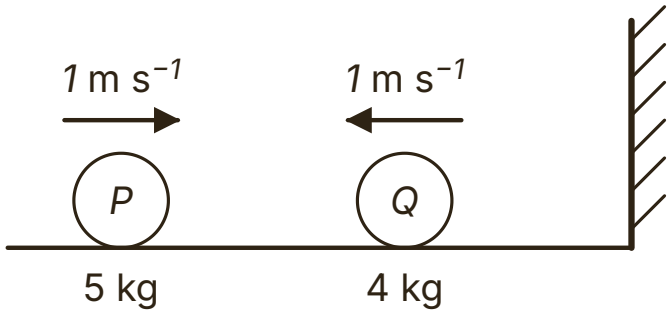
D1

> Alex writes: "After Q hits the wall, its velocity is e times its velocity before the impact."

Smooth spheres P and Q of equal radius have masses $5\ \text{kg}$ and $4\ \text{kg}$. They move towards each other, each at $1\ \text{m s}^{-1}$, along a line that is perpendicular to a smooth vertical wall, with Q between P and the wall. The coefficient of restitution between P and Q, and between Q and the wall, is e.

(a) Show that, after the first collision, Q moves towards the wall with speed $\dfrac{1+10e}{9}\ \text{m s}^{-1}$. [3 marks]



(b) Find, in terms of e, the velocity of P after the first collision. [2 marks]

Answer .....

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(c) Explain what is wrong with Alex's statement, and write down the velocity of Q after it hits the wall. [2 marks]

Answer .....

(d) Find the set of values of e for which P and Q collide again. [4 marks]

Answer .....

Total for D1: 11 marks

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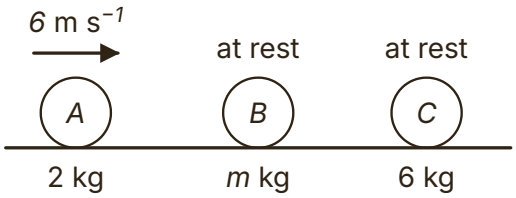
D2 Three smooth spheres A, B and C of equal radius lie in a straight line on a smooth horizontal surface, with B between A and C. A has mass $2\ \text{kg}$, B has mass $m\ \text{kg}$ and C has mass $6\ \text{kg}$. B and C are at rest. A is projected towards B at $6\ \text{m s}^{-1}$. The coefficient of restitution between A and B is $\frac{1}{2}$. After A and B collide, A is at rest.

(a) Show that $m = 4$. [3 marks]



(b) B then hits C. The coefficient of restitution between B and C is e. Find, in terms of e, the velocities of B and C after this collision. [3 marks]

Answer .....

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(c) Find the set of values of e for which B collides with A again. [3 marks]

Answer .....

(d) Given that $e = 1$, find the total kinetic energy lost in the two collisions. [2 marks]

Answer .....

Total for D2: 11 marks

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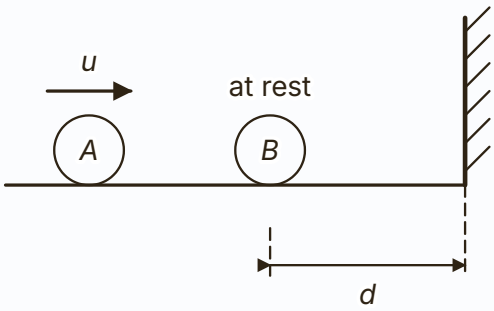
E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Smooth spheres A and B of equal radius and equal mass lie on a smooth horizontal surface. B is at rest at a distance d from a smooth vertical wall that is perpendicular to the line of centres. A moves towards B with speed u. The coefficient of restitution between A and B, and between B and the wall, is e, where $0 < e \leqslant 1$. Treating the spheres as particles, find, in terms of d and e, the distance from the wall at which A and B collide for the second time. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Find the velocities of A and B after the first collision, taking the direction towards the wall as positive.

Hint 2 Find how long B takes to reach the wall, and where A is at that moment.

Hint 3 After B rebounds the two spheres close the gap at the sum of their speeds. B's distance from the wall is the distance it travels after the rebound.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the rebound velocity and the impulse when a sphere hits a smooth wall at right angles.	○ ○ ○
I can work through a chain of collisions one impact at a time, with one positive direction.	○ ○ ○
I can decide whether a further collision happens by comparing velocities.	○ ○ ○
I can find the values of e, or of a mass ratio, for which a further collision happens.	○ ○ ○

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