Newton's experimental law and the coefficient of restitution

91 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-restitution/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Sphere A of mass $2\ \text{kg}$ moves at $6\ \text{m s}^{-1}$ and hits sphere B of mass $3\ \text{kg}$, which is at rest. Afterwards A moves at $1.5\ \text{m s}^{-1}$ in its original direction. Find the velocity of B.

Answer .....

A2 Needed today [2 marks]

Solve $3x + y = 10$ and $y - x = 2$.

Answer .....

A3 Needed today [2 marks]

Find the values of e, with $0 \leqslant e \leqslant 1$, for which $2 - 3e < 0$.

Answer .....

A4 Last lesson [2 marks]

A ball of mass $0.5\ \text{kg}$ moving at $4\ \text{m s}^{-1}$ is hit back along the same line at $2\ \text{m s}^{-1}$. Find the magnitude of the impulse on it.

Answer .....

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Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

PART 1 Finding the coefficient of restitution

EXAMPLE 1 · I DO

Smooth spheres A and B of equal radius move in the same direction along a straight line, A at $6\ \mathrm{m\,s^{-1}}$ and B at $2\ \mathrm{m\,s^{-1}}$. A catches up with B. After the collision A moves at $3\ \mathrm{m\,s^{-1}}$ and B at $5\ \mathrm{m\,s^{-1}}$, both in the original direction. Find the coefficient of restitution.

Hint Newton's experimental law: speed of separation $= e\times$ speed of approach. Approach is how fast the gap closes before; separation is how fast it opens after.

YOUR TURN · WE DO

B1 Smooth spheres A and B of equal radius move towards each other along a straight line, A at $5\ \mathrm{m\,s^{-1}}$ and B at $3\ \mathrm{m\,s^{-1}}$. After the collision both directions are reversed: A moves at $1\ \mathrm{m\,s^{-1}}$ and B at $2\ \mathrm{m\,s^{-1}}$. Find the coefficient of restitution. [3 marks]

Answer .....

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C1 Spheres A and B move towards each other, A at $3\ \mathrm{m\,s^{-1}}$ and B at $2\ \mathrm{m\,s^{-1}}$. After the collision A moves back at $2.5\ \mathrm{m\,s^{-1}}$ and B moves back at $1.5\ \mathrm{m\,s^{-1}}$. Sam writes: [4 marks]

$$e = \frac{3 + 2}{2.5 + 1.5} = 1.25.$$

Give two reasons why Sam must be wrong, and find the correct value of e.

Answer .....

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C2 Sphere A of mass $1\ \mathrm{kg}$ moves at $8\ \mathrm{m\,s^{-1}}$. Sphere B, of equal radius and mass $4\ \mathrm{kg}$, moves at $1\ \mathrm{m\,s^{-1}}$ in the same direction. A catches B and after the collision A is at rest. Find the velocity of B and the coefficient of restitution. [4 marks]

Answer .....

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PART 2 Finding the velocities after a collision

EXAMPLE 2 · I DO

Sphere A of mass $2\ \text{kg}$ moves at $5\ \text{m s}^{-1}$. Sphere B, of equal radius and mass $3\ \text{kg}$, moves at $1\ \text{m s}^{-1}$ in the same direction. A catches B and the coefficient of restitution is 0.5. Find the velocities of A and B after the collision.

Hint Call the velocities after v_A and v_B, both in the original direction. Write one momentum equation and one restitution equation, then solve them together.

YOUR TURN · WE DO

B2 Sphere A of mass $3\ \text{kg}$ moves at $6\ \text{m s}^{-1}$ and hits sphere B, of equal radius and mass $1\ \text{kg}$, which is at rest. The coefficient of restitution is 0.6. Find the velocities of A and B after the collision. [5 marks]

Answer .....

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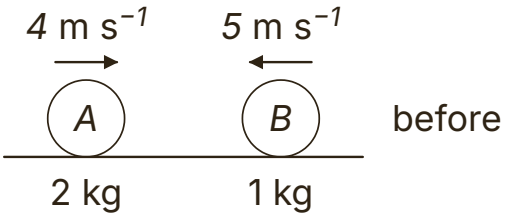
EXAMPLE 3 · I DO

Sphere A of mass $2\ \text{kg}$ moves at $4\ \text{m s}^{-1}$. Sphere B, of equal radius and mass $1\ \text{kg}$, moves at $5\ \text{m s}^{-1}$ towards A. The coefficient of restitution is 0.4. Find the velocities of A and B after the collision.



before

Hint Take A's direction as positive, so B's velocity before is -5. The speed of approach is $4-(-5)$.

YOUR TURN · WE DO

B3 Sphere A of mass $3\ \text{kg}$ moves at $2\ \text{m s}^{-1}$. Sphere B, of equal radius and mass $2\ \text{kg}$, moves at $4\ \text{m s}^{-1}$ towards A. The coefficient of restitution is 0.5. Find the speed of each sphere after the collision and state its direction. [6 marks]

Answer .....

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C3 Spheres A and B, of equal radius, move towards each other, A at $7\ \mathrm{m\,s^{-1}}$ and B at $3\ \mathrm{m\,s^{-1}}$. The coefficient of restitution is 0.5. After the collision B moves at $4\ \mathrm{m\,s^{-1}}$ with its direction reversed. Find the velocity of A after the collision, and find the ratio of the mass of A to the mass of B. [5 marks]

Answer .....

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C4 Sphere A of mass $0.5\ \text{kg}$ moves at $6\ \text{m s}^{-1}$. Sphere B, of equal radius and mass $0.3\ \text{kg}$, moves at $2\ \text{m s}^{-1}$ in the same direction. A catches B and the coefficient of restitution is 0.5. Find the velocities of A and B after the collision, and the magnitude of the impulse on B. [6 marks]

Answer .....

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PART 3 Perfectly elastic, inelastic and a range of e

EXAMPLE 4 · I DO

Smooth spheres A and B have equal mass and equal radius. A moves at $4\ \mathrm{m\,s^{-1}}$ and hits B, which is at rest. Find their velocities after the collision when (i) $e = 1$, (ii) $e = 0$. Name each type of collision.

Hint $e = 1$: they separate as fast as they approached. $e = 0$: they do not separate at all.

YOUR TURN · WE DO

B4 Sphere A of mass $1\ \mathrm{kg}$ moves at $6\ \mathrm{m\,s^{-1}}$ and hits sphere B, of equal radius and mass $2\ \mathrm{kg}$, which is at rest. Find the velocities of A and B after the collision when (i) the collision is perfectly elastic, (ii) the collision is inelastic. [5 marks]

Answer .....

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EXAMPLE 5 · I DO

Sphere A of mass $1\ \text{kg}$ moves at $9\ \text{m s}^{-1}$ and hits sphere B, of equal radius and mass $2\ \text{kg}$, which is at rest. The coefficient of restitution is e. Find the velocities of A and B after the collision in terms of e. Find the range of values of e for which A changes direction.

Hint Solve the two equations with e left as a letter. A changes direction when its velocity after is negative. Remember that $0 \leqslant e \leqslant 1$.

YOUR TURN · WE DO

B5 Sphere A of mass $2\ \text{kg}$ moves at $5\ \text{m s}^{-1}$ and hits sphere B, of equal radius and mass $3\ \text{kg}$, which is at rest. The coefficient of restitution is e. Find the velocities of A and B after the collision in terms of e, and the range of values of e for which A changes direction. [6 marks]

Answer .....

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C5 Sphere A of mass $2\ \text{kg}$ moves at $5\ \text{m s}^{-1}$ and hits sphere B, of equal radius and mass $3\ \text{kg}$, which is at rest. After the collision A moves at $1\ \text{m s}^{-1}$ with its direction reversed. Find the velocity of B, and the coefficient of restitution. Name this type of collision. [5 marks]

Answer .....

C6 Two smooth spheres A and B have equal mass and equal radius. A moves with speed u and hits B, which is at rest. The coefficient of restitution is e.

(a) Show that after the collision A moves with speed $\frac{1}{2}u(1-e)$, and find the speed of B. [4 marks]

Answer .....

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(b) The speed of A after the collision is one fifth of the speed of B. Find e. [3 marks]

Answer .....

(c) Explain why A can never change direction in this collision, whatever the value of e. [2 marks]

Total for C6: 9 marks

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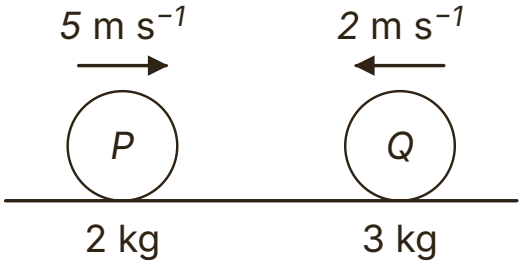
D1 Sphere P of mass $2\ \text{kg}$ moves at $5\ \text{m s}^{-1}$. Sphere Q, of equal radius and mass $3\ \text{kg}$, moves at $2\ \text{m s}^{-1}$ towards P. The coefficient of restitution between them is e.

(a) Show that the velocity of Q after the collision is $\dfrac{4+14e}{5}\ \text{m s}^{-1}$ in the direction P was first moving. [4 marks]



before

(b) Find the velocity of P after the collision in terms of e. [2 marks]

Answer .....

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(c) Find the range of values of e for which P changes direction. [3 marks]

Answer .....

Total for D1: 9 marks

D2

> Alex says: "The spheres bounced apart, so the collision is perfectly elastic."

Sphere A of mass $4\ \text{kg}$ moves at $3\ \text{m s}^{-1}$ and hits sphere B, of equal radius and mass $2\ \text{kg}$, which is at rest. After the collision A moves at $1.5\ \text{m s}^{-1}$ in its original direction. The spheres do not stick together.

(a) Find the velocity of B after the collision. [2 marks]

Answer .....

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(b) Find the coefficient of restitution. [2 marks]

Answer .....

(c) Explain why Alex is wrong. [2 marks]

(d) The spheres are in contact for 0.05 s. Find the magnitude of the average force on B. [2 marks]

Answer .....

Total for D2: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Sphere A of mass $2m$ moves with speed u. Sphere B, of equal radius and mass m, moves with speed ku towards A, where $k > 0$. The coefficient of restitution is e. Find the range of values of k for which it is possible for A to be brought to rest by the collision. For $k = 1$, find the value of e that brings A to rest. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write v_A in terms of e, k and u, with B's velocity before as $-ku$.

Hint 2 Set $v_A = 0$ and make e the subject.

Hint 3 e must lie between 0 and 1 inclusive. Turn each end of that into an inequality in k.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the coefficient of restitution from speeds of approach and separation.	○ ○ ○
I can use conservation of momentum and Newton's experimental law together to find velocities.	○ ○ ○
I can find the range of values of e for which a sphere changes direction.	○ ○ ○
I can say what $e = 1$ and $e = 0$ mean for a collision.	○ ○ ○

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