Motion in a vertical circle

93 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-mech-vertical-circles/

Date

A Do Now

Quick questions to start.

A1 From chapter 2 [2 marks]

A particle moving at $1\ \mathrm{m\,s^{-1}}$ slides down a smooth curved slope, falling a vertical height of $2.4\ \mathrm{m}$. Find its speed at the bottom.

Answer .....

A2 Needed today [2 marks]

A string of length $0.6\ \mathrm{m}$ hangs from a fixed point O and makes an angle of $50°$ with the downward vertical. Find the height of its lower end above the lowest point it can reach.

Answer .....

A3 Needed today [1 mark]

A particle moves on a circle of radius $0.5\ \mathrm{m}$ with speed $3\ \mathrm{m\,s^{-1}}$. Find the size of its acceleration towards the centre.

Answer .....

A4 From A-Level Maths [2 marks]

Solve $2c^2 + 5c - 3 = 0$. One root can be the value of $\cos\theta$ and one cannot: say which, and why.

Answer .....

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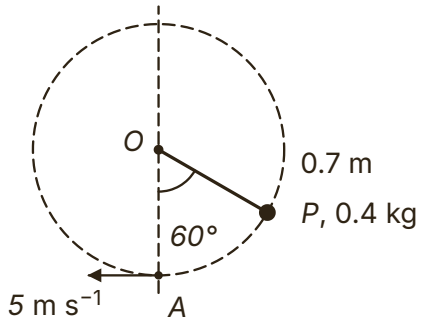
Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

PART 1 Speed and tension at a point of the circle

EXAMPLE 1 · I DO

A particle P of mass $0.4\ \text{kg}$ hangs from a fixed point O on a light inextensible string of length $0.7\ \text{m}$. It is projected horizontally from its lowest point A with speed $5\ \text{m s}^{-1}$ and moves in a vertical circle. Find the speed of P and the tension in the string when OP makes an angle of 60° with the downward vertical.



Hint Energy for the speed: P has risen $0.7 - 0.7\cos 60^\circ$, not $0.7\cos 60^\circ$. Then Newton's second law along PO: the weight has a component $mg\cos 60^\circ$ away from O, so $T - mg\cos 60^\circ = \dfrac{mv^2}{r}$.

YOUR TURN · WE DO

B1 A particle of mass $0.25\ \text{kg}$ on a light inextensible string of length $1.1\ \text{m}$ is projected from its lowest point with speed $6\ \text{m s}^{-1}$ and moves in a vertical circle about the fixed end O. Find the speed and the tension when the string makes 40° with the downward vertical. [4 marks]

Answer .....

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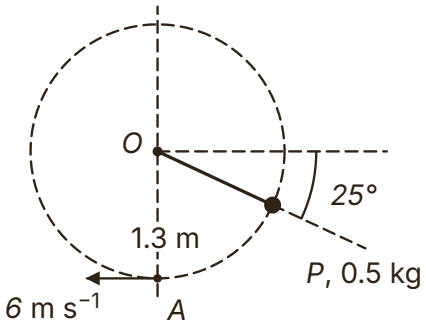
C1 A particle of mass $0.5\ \text{kg}$ on a light inextensible string of length $1.3\ \text{m}$ is projected from its lowest point with speed $6\ \text{m s}^{-1}$. Find its speed and the tension at the moment the string, still below the level of O, makes an angle of 25° with the horizontal. [5 marks]



Answer .....

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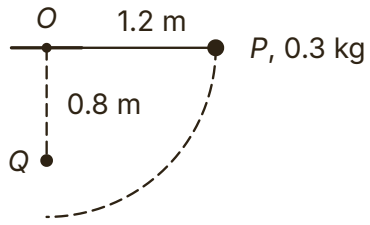
C2 A particle P of mass $0.3\ \text{kg}$ hangs on a light inextensible string of length $1.2\ \text{m}$ from a fixed point O. A small smooth peg Q is fixed $0.8\ \text{m}$ vertically below O. P is released from rest with the string taut and horizontal. When the string reaches the vertical it catches on the peg. Find the tension just before and just after the string meets the peg, and decide whether P then makes complete circles about Q. [6 marks]



Answer .....

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PART 2 Complete circles on a string, a rod or a wire

EXAMPLE 2 · I DO

A particle is projected from the lowest point of a vertical circle of radius $0.85\ \text{m}$. Find the least speed of projection for it to make complete circles (a) on a light inextensible string fixed at the centre, (b) as a bead threaded on a smooth circular wire.

Hint A string can go slack, so at the top it needs $T \geq 0$: $\dfrac{mv^2}{r} = T + mg \geq mg$, so $v^2 \geq gr$ there. A bead on a wire cannot leave it, so it only needs $v \geq 0$ at the top. Then energy from the bottom, rising $2r$.

YOUR TURN · WE DO

B2 A bead of mass $0.6\ \text{kg}$ is threaded on a smooth circular wire. The wire has radius $1.4\ \text{m}$ and is fixed in a vertical plane. Find the least speed with which it must be projected from the lowest point to go all the way round, and the force from the wire on the bead at the lowest point when projected at this speed. [4 marks]

Answer .....

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EXAMPLE 3 · I DO

A small ball of mass $0.5\ \mathrm{kg}$ is fixed to one end of a light rod of length $0.6\ \mathrm{m}$. The other end is freely pivoted at O, and the ball moves in a vertical circle. It passes through the lowest point with speed $5\ \mathrm{m\,s^{-1}}$. Find its speed at the highest point and the force in the rod there, stating whether it is a tension or a thrust.

Hint Take the rod force as a tension T towards O. At the top both T and the weight point towards O: $T + mg = \dfrac{mv^2}{r}$. A negative T means the rod pushes: a thrust.

YOUR TURN · WE DO

B3 A ball of mass $0.2\ \mathrm{kg}$ on a light rod of length $0.75\ \mathrm{m}$, freely pivoted at O, passes through the lowest point with speed $6.5\ \mathrm{m\,s^{-1}}$. Find its speed at the highest point and the force in the rod there, stating whether it is a tension or a thrust. [4 marks]

Answer .....

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C3 A particle of mass $0.4\ \text{kg}$ on a light inextensible string of length $0.6\ \text{m}$ makes complete vertical circles with the least possible speed. Find its speed at the highest point and the tension in the string at the lowest point. [4 marks]

Answer .....

C4 A particle on a light inextensible string of length $1\ \text{m}$ moves in a vertical circle about the fixed end. Sam writes: “To just get round, its speed at the top must be 0. Energy gives $u^2 = 4g(1)$, so the least speed at the bottom is $6.26\ \text{m s}^{-1}$.” Explain Sam's mistake, and find the correct least speed at the bottom. [5 marks]

Answer .....

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C5 A bead P is threaded on a smooth circular wire of radius $0.5\ \mathrm{m}$ fixed in a vertical plane with centre O. It is projected from the lowest point with speed $4\ \mathrm{m\,s^{-1}}$. Show that it does not reach the highest point. Find the angle between OP and the downward vertical when the bead first comes to rest. [5 marks]

Answer .....

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PART 3 Where a string goes slack or a particle leaves a surface

EXAMPLE 4 · I DO

A particle on a light inextensible string of length $0.5\ \text{m}$ is projected from its lowest point with speed $4.5\ \text{m s}^{-1}$. Show that it does not make complete circles. Find the angle the string makes with the upward vertical when it goes slack, the speed of the particle then, and its height above the lowest point.

Hint It needs $u^2 \geq 5gr = 24.5$. The string goes slack where $T = 0$. Let the string make ϕ with the upward vertical: then energy gives v^2 in terms of ϕ, and along the string $T + mg\cos\phi = \dfrac{mv^2}{r}$. Put $T = 0$.

YOUR TURN · WE DO

B4 A particle on a light inextensible string of length $0.8\ \text{m}$ is projected from its lowest point with speed $5.6\ \text{m s}^{-1}$. Find the angle between the string and the upward vertical when the string goes slack, the speed then, and the height above the lowest point. [5 marks]

Answer .....

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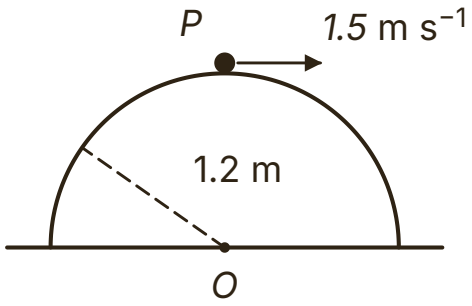
EXAMPLE 5 · I DO

A smooth solid hemisphere of radius $1.2\ \mathrm{m}$ is fixed with its flat face on horizontal ground. A particle at the top of the hemisphere is set moving with speed $1.5\ \mathrm{m\,s^{-1}}$. Find the angle between OP and the upward vertical, where O is the centre, when the particle leaves the surface, and its speed then.



Hint Use a separate angle θ for a general point. The particle falls $1.2 - 1.2\cos\theta$. Towards O: $mg\cos\theta - R = \dfrac{mv^2}{a}$. It leaves where $R = 0$.

YOUR TURN · WE DO

B5 A particle at the top of a smooth solid hemisphere of radius $0.9\ \mathrm{m}$, fixed with its flat face on the ground, is set moving with speed $1\ \mathrm{m\,s^{-1}}$. Find the angle between OP and the upward vertical when it leaves the surface, and its speed then. [4 marks]

Answer .....

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C6 A particle P of mass $0.3\ \text{kg}$ hangs at rest from a fixed point O on a light inextensible string of length $0.75\ \text{m}$. It is given a horizontal impulse of magnitude $1.5\ \text{N s}$. Find the angle between the string and the upward vertical when the string goes slack, and the speed of P then. [4 marks]

Answer .....

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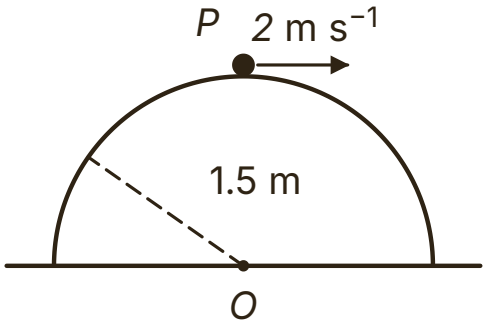
C7 In this question you must show detailed reasoning. A smooth solid hemisphere of radius $1.5\ \mathrm{m}$ is fixed with its flat face on horizontal ground, and has centre O. A particle P is projected horizontally from the top of the hemisphere with speed $2\ \mathrm{m\,s^{-1}}$. Find the angle between OP and the upward vertical when P leaves the surface, the speed of P then, and its height above the ground. Find also the greatest speed with which P could be projected from the top so that it moves on the surface at first. [8 marks]



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Answer .....

D Exam-style questions

OCR exam style

D1

> Alex says: "The ball needs a speed of at least $\sqrt{5g \times 0.8}$ at the lowest point to get over the top, the same as on a string."

A small ball of mass $0.5\ \text{kg}$ is fixed to one end of a light rod of length $0.8\ \text{m}$. The other end of the rod is freely pivoted at a fixed point O, and the ball moves in a vertical circle.

(a) Explain why Alex is wrong. [2 marks]

(b) Find the least speed at the lowest point for the ball to make complete circles. [2 marks]

Answer .....

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(c) The ball passes through the lowest point with speed $6\ \mathrm{m\,s^{-1}}$. Find its speed at the highest point. [2 marks]

Answer .....

(d) Find the force in the rod when the ball is at the highest point, stating whether it is a tension or a thrust. [3 marks]

Answer .....

Total for D1: 9 marks

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D2 A particle P of mass m is attached to one end of a light inextensible string of length a, whose other end is fixed at O. P is projected horizontally from the point vertically below O with speed u, and moves in a vertical circle.

(a) Show that, when OP makes an angle θ with the downward vertical, the tension in the string is $\frac{mu^2}{a} + mg(3\cos\theta - 2)$. [3 marks]

(b) Hence show that P makes complete circles if and only if $u^2 \geq 5ga$. [2 marks]

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(c) In the case $u^2 = 3.5ga$, find the value of θ at which the string goes slack, and the speed of P then, in terms of g and a. [3 marks]

Answer .....

(d) Find, in terms of a, how far above O the particle is when the string goes slack. [2 marks]

Answer .....

Total for D2: 10 marks

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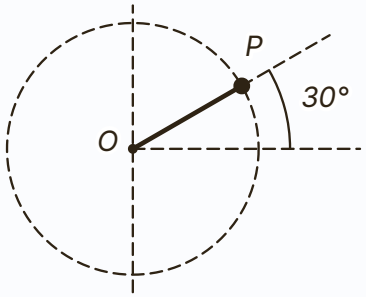
E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 In this question you must show detailed reasoning. A small ball P of mass m is fixed to one end of a light rod of length $1.2\ \text{m}$, whose other end is freely pivoted at a fixed point O. P is released from rest with the rod above the level of O, making 30° with the horizontal, and swings down in a vertical circle. Let θ be the angle between OP and the downward vertical. Show that the force in the rod, taken as a tension, is $mg(3\cos\theta + 1)$. Hence find the value of θ at which P is moving downwards fastest, and its downward speed then. [9 marks]



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Start as always: the speed from energy (how far has P fallen from $\theta = 120°$?), then Newton's second law along PO.

Hint 2 The downward speed is greatest when the downward acceleration is zero: the forces on P have no vertical resultant there.

Hint 3 You get a quadratic in $\cos\theta$. One root is a point P never reaches: say why, and reject it. The velocity is along the tangent, so its vertical part is $v\sin\theta$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the speed at any point of a vertical circle by energy, using the height $r(1-\cos\theta)$.	○ ○ ○
I can find the tension or contact force by Newton's second law towards the centre, including the weight component.	○ ○ ○
I can find the least speed for complete circles on a string, a rod or a wire, and say why they differ.	○ ○ ○
I can find where a string goes slack or a particle leaves a surface, and reject an impossible root.	○ ○ ○

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