A-LEVEL FURTHER MATHS (OCR A) · FURTHER MECHANICS · CHAPTER 1: DIMENSIONAL ANALYSIS · A* EXAM PRACTICE

Hints online, answers free with an account:
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Dimensional analysis: A* questions

80 marks

Date

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X A* exam questions

A to A*

Extended OCR-style questions. Show every step. The hints are at the end of the sheet. Use one at a time.

X1 In this question you must show detailed reasoning. [8 marks]

A small bead of mass m is threaded on a smooth circular wire of radius r. The wire is fixed in a vertical plane. The bead is projected from the lowest point of the wire with speed u.

The least value of u for which the bead reaches the highest point of the wire is modelled as

$$u = k\, m^{\alpha} r^{\beta} g^{\gamma},$$

where k is a dimensionless constant.

Use dimensional analysis to find α, β and γ. Then, by considering energy and without quoting a result for motion in a vertical circle, find the value of k.

Answer $\alpha = \ldots$, $\beta = \ldots$, $\gamma = \ldots$, $k = \ldots$

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X2 In this question you must show detailed reasoning. [10 marks]

Water of density ρ comes out of a hose with cross-sectional area A. It travels horizontally at speed v and hits a vertical wall at right angles. The water then moves directly away from the wall with speed ev, where e is a constant with $0 \leqslant e \leqslant 1$. Assume the stream does not spread out before it reaches the wall.

The force F that the water exerts on the wall is modelled as

$$F = k\,\rho^{\alpha} A^{\beta} v^{\gamma},$$

where k is dimensionless and may depend on e.

Use dimensional analysis to find α, β and γ. By considering the water that reaches the wall in a time t, find k in terms of e.

Hence find F when $\rho = 1000\ \text{kg}\,\text{m}^{-3}$, $A = 2.5\ \text{cm}^2$, $v = 14\ \text{m}\,\text{s}^{-1}$ and $e = 0.25$.

Answer $\alpha = \ldots$, $\beta = \ldots$, $\gamma = \ldots$, $k = \ldots$, $F = \ldots$ N

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X3 Two particles P and Q, of masses m_1 and m_2, move towards each other along a straight line with speeds u_1 and u_2. They collide directly, and the coefficient of restitution between them is e. A textbook gives the loss of kinetic energy in the collision as

$$\Delta E = \frac{m_1 m_2 (1 - e^2)(u_1 + u_2)^2}{2(m_1 + m_2)}.$$

(a) Show that this formula is dimensionally consistent. Explain why $m_1 + m_2$ and $1 - e^2$ have the dimensions you use for them. [3 marks]

(b) In one collision $m_1 = 2\ \text{kg}$, $u_1 = 4\ \text{m s}^{-1}$, $m_2 = 3\ \text{kg}$, $u_2 = 1\ \text{m s}^{-1}$ and $e = 0.5$. By finding the velocities of P and Q after the collision, show that the formula gives the correct loss of kinetic energy. [5 marks]

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(c) A student copies the formula with $(1-e)^2$ in place of $(1-e^2)$. Explain why dimensional analysis cannot show that the student's formula is wrong, and use your answer to part (b) to show that it is wrong. [2 marks]

Answer .....

Total for X3: 10 marks

X4 A car of mass m moves along a straight level road. Its engine works at a constant power P. The resistance to its motion is cv, where v is its speed and c is a constant. The greatest possible speed of the car is V.

(a) Show that $V = \sqrt{\dfrac{P}{c}}$, stating the condition you use. [2 marks]

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(b) The car starts from rest. The time τ it takes to reach the speed $\frac{1}{2}V$ is modelled as $\tau = k\, m^{\alpha} P^{\beta} c^{\gamma}$, where k is a dimensionless constant. Find the dimensions of c, and use dimensional analysis to find α, β and γ. [5 marks]

Answer $\alpha = \ldots$, $\beta = \ldots$, $\gamma = \ldots$

(c) Car B has the same value of c as car A, 1.5 times the mass of car A and an engine 4 times as powerful. According to the model, find the ratio of B's greatest speed to A's, and the ratio of the time B takes to reach half its own greatest speed to the time A takes to reach half its own greatest speed. [3 marks]

Answer $V_B : V_A = \ldots$, $\tau_B : \tau_A = \ldots$

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(d) A student wants to include V in the model as well as m, P and c. Explain why V is not needed. [2 marks]

Answer .....

Total for X4: 12 marks

X5 A small block of mass m is projected with speed u along rough horizontal ground. The coefficient of friction between the block and the ground is μ. The block slides a distance d before it comes to rest.

(a) The distance is modelled as $d = k\, m^{\alpha} u^{\beta} g^{\gamma}$, where k is dimensionless. Use dimensional analysis to find α, β and γ. Explain why dimensional analysis cannot show how d depends on μ. [4 marks]

Answer $\alpha = \ldots$, $\beta = \ldots$, $\gamma = \ldots$

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(b) Use the work-energy principle to find k in terms of μ. [3 marks]

Answer $k = \ldots$

(c) The block is now projected with speed $6\ \mathrm{m\,s^{-1}}$ up a line of greatest slope of a rough plane inclined at θ to the horizontal, where $\sin\theta = 0.28$ and $\cos\theta = 0.96$. The coefficient of friction is 0.25. A student says that the block travels $\dfrac{u^2}{2\mu g}$ up the plane, because dimensional analysis gives the same powers of u and g on a slope. Find how far the block actually travels up the plane, and explain what is wrong with the student's reasoning. [3 marks]

Answer $d = \ldots\ \mathrm{m}$

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(d) Determine whether the block then slides back down the plane. [2 marks]

Answer .....

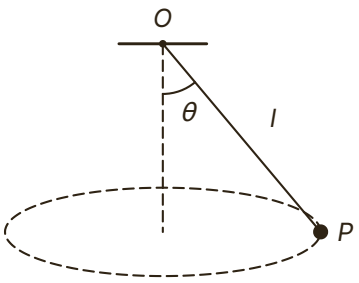
Total for X5: 12 marks

X6 A particle P of mass m is attached to one end of a light inextensible string of length l. The other end is attached to a fixed point O. P moves in a horizontal circle with constant angular speed ω, with the string at an angle θ to the downward vertical. A student models the tension T in the string as

$$T = k\, m^{\alpha} g^{\beta} l^{\gamma} \omega^{\delta},$$

where k is a dimensionless constant.

(a) Explain why dimensional analysis alone cannot find all four powers. [2 marks]



Answer .....

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(b) Find α, and find β and γ in terms of δ. [4 marks]

Answer $\alpha = \ldots$, $\beta = \ldots$, $\gamma = \ldots$

(c) By resolving horizontally, show that $T = m\omega^2 l$. Hence find δ and k, and explain why g does not appear in the formula for T even though P has weight. [4 marks]

Answer $\delta = \ldots$, $k = \ldots$

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(d) The string has length $0.6\ \mathrm{m}$ and $\omega = 5\ \mathrm{rad\,s^{-1}}$. The angular speed is then doubled and the string is shortened to half its length. Find the factor by which the tension changes, and find the angle between the string and the vertical before and after the change. [4 marks]

Answer factor ... , before ...° , after ...°

Total for X6: 14 marks

X7 A crane lifts a load of mass m from rest on the ground to rest at a height h. The engine of the crane can work at a power of at most P. The least time τ for the lift is modelled as

$$\tau = k\,P^{a}m^{b}h^{c}g^{d},$$

where k is a dimensionless constant.

(a) Use dimensional analysis to find a, b and d in terms of c. [4 marks]

Answer $a = \ldots$, $b = \ldots$, $d = \ldots$

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(b) The crane lifts the same load through $15\ \mathrm{m}$ in a least time of $11.0\ \mathrm{s}$, and through $36\ \mathrm{m}$ in a least time of $26.4\ \mathrm{s}$. Use these results to find c, and write down the formula for τ. [3 marks]

Answer $c = \ldots$, $\tau = \ldots$

(c) The load has mass $450\ \mathrm{kg}$ and the greatest power of the engine is $7.2\ \mathrm{kW}$. Find k. [2 marks]

Answer $k = \ldots$

(d) Use an energy argument to show that, for any crane and load described by this model, k cannot be less than 1. [3 marks]

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(e) Give two reasons why k is greater than 1 for the real crane. [2 marks]

Answer .....

Total for X7: 14 marks

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find unknown powers by dimensional analysis and then find the constant from first principles	○	○	○
I can explain why a quantity is not needed, or why a dimensionless factor cannot be found	○	○	○
I can use data or a physical argument to fix a power when there are more unknowns than equations	○	○	○
I can check a formula against a real calculation and criticise the model behind it	○	○	○

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Hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

X1	**Hint 1**	Write the dimensions of u, m, r and g in terms of M, L and T, and match the powers of each.
	Hint 2	For k, write an energy equation between the lowest point and the highest point of the wire.
	Hint 3	The bead is threaded on the wire, so the wire can push it either way. Ask what the least speed at the top can be.
X2	**Hint 1**	Density is mass per unit volume, so $[\rho] = \mathrm{ML}^{-3}$. Match the powers of M, L and T.
	Hint 2	In time t, a length vt of the stream reaches the wall. Find its mass and its change in momentum.
	Hint 3	The water comes back, so its change in velocity is $v + ev$, not $v - ev$. Force is change in momentum divided by time. Change cm^2 to m^2.
X3	**Hint 1**	For (a), you may only add quantities with the same dimensions. What are the dimensions of e?
	Hint 2	For (b), use conservation of momentum and Newton's experimental law, with one direction taken as positive. Q's velocity before is $-1\ \mathrm{m\,s^{-1}}$.
	Hint 3	For (c), a dimensional check cannot see any number or dimensionless factor. Put $e = 0.5$ into the student's version and compare with (b).
X4	**Hint 1**	For (a), what is the acceleration at the greatest speed? Power is driving force times speed.
	Hint 2	For (b), find $[c]$ from $[cv] = [\mathrm{force}]$. Then match the powers of M, L and T; the L equation alone fixes one power.
	Hint 3	For (c) and (d), look at which quantities appear in τ and in V. Part (a) tells you V in terms of P and c.
X5	**Hint 1**	For (a), match the powers of M, L and T. What are the dimensions of $\mu = \dfrac{F}{R}$?
	Hint 2	For (b) and (c), the work done against friction is friction times distance. On the slope the normal reaction is $mg\cos\theta$ and the block also gains height.
	Hint 3	For (d), compare the pull of gravity down the slope with the greatest friction the surface can give.

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X6	**Hint 1**	For (a) and (b), each of M, L and T gives one equation. Count the unknowns. $[\omega] = \mathrm{T}^{-1}$.
	Hint 2	For (c), the particle moves in a circle of radius $l\sin\theta$, not l. Resolve horizontally towards the centre.
	Hint 3	For (d), use $T = m\omega^2 l$ for the factor, and resolve vertically for the angle.
X7	**Hint 1**	For (a), match the powers of M, L and T. There are three equations, so leave c as the free power.
	Hint 2	For (b), only h changes between the two lifts. Compare the ratio of the times with the ratio of the heights.
	Hint 3	For (d), compare the least work the engine must do with the most work it can do in time τ. Change kW to W in (c).

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