Modulus-argument form: multiplying and dividing

87 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-modarg-form/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the modulus of $-8 + 15i$, and its argument to 3 significant figures.

Answer .....

A2 Needed today [2 marks]

Write down the exact values of $\cos \frac{2\pi}{3}$ and $\sin\left(-\frac{\pi}{4}\right)$.

Answer .....

A3 From A-Level Maths [2 marks]

Write down the expansions of $\cos(A + B)$ and $\sin(A + B)$.

Answer .....

A4 Last lesson [2 marks]

Write down $\arg(-5i)$ and $\arg(-2)$, in the interval $(-\pi, \pi]$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 From modulus-argument form to a + bi

EXAMPLE 1 · I DO

Write in the form $a + bi$ (i) $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$, exactly (ii) $6(\cos 2.5 + i\sin 2.5)$, to 3 significant figures.

Hint Multiply out: $a = r\cos\theta$ and $b = r\sin\theta$. The form $r(\cos\theta + i\sin\theta)$ is also written $[r, \theta]$, with square brackets.

YOUR TURN · WE DO

B1 Write in the form $a + bi$ (i) $8\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$, exactly (ii) $5(\cos(-2.1) + i\sin(-2.1))$, to 3 significant figures. [4 marks]

Answer .....

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C1 Write in the form $a + bi$ (i) $\left[6, -\frac{5\pi}{6}\right]$, exactly (ii) $[3, 4]$, to 3 significant figures. [4 marks]

Answer .....

C2 Sam is asked to write $5(\cos 2.3 + i \sin 2.3)$ in the form $x + yi$. He solves $x^2 + y^2 = 25$ and $\frac{y}{x} = \tan 2.3$, takes the positive square root for x, and gets $3.33 - 3.73i$. Explain why Sam's answer cannot be right, and find the correct answer to 3 significant figures. [4 marks]

Answer .....

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PART 2 Writing a complex number in modulus-argument form

EXAMPLE 2 · I DO

Write $-2 + 2\sqrt{3}i$ in modulus-argument form, with the argument exact.

Hint Find r first. Sketch the point to see which quadrant it is in.

YOUR TURN · WE DO

B2 Write in modulus-argument form, with the argument exact (i) $-3\sqrt{3} - 3i$ (ii) $5\sqrt{2} - 5\sqrt{2}i$. [4 marks]

Answer .....

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EXAMPLE 3 · I DO

Write $-7 + 24i$ in modulus-argument form, with the argument to 3 significant figures.

Hint The angle with the negative real axis is $\tan^{-1}\frac{24}{7}$.

YOUR TURN · WE DO

B3 $z = 20 - 21i$. Write z in modulus-argument form with the argument in $(-\pi, \pi]$, and again with the argument in $[0, 2\pi)$. Give arguments to 3 significant figures. [4 marks]

Answer .....

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C3 Write in modulus-argument form, with the argument exact (i) $\sqrt{3} - i$ (ii) $-6 - 6i$. [4 marks]

Answer .....

C4 $z = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right)$. Write z^*, $-z$ and $\frac{1}{z}$ in modulus-argument form, with arguments in $(-\pi, \pi]$. [5 marks]

Answer .....

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PART 3 Multiplying and dividing in modulus-argument form

EXAMPLE 4 · I DO

$z = 3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ and $w = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$. Find zw and $\frac{z}{w}$ in modulus-argument form.

Hint To multiply, multiply the moduli and add the arguments. To divide, divide the moduli and subtract the arguments.

YOUR TURN · WE DO

B4 $z = 4\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$ and $w = 8\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$. Find zw in the form $a + bi$, and $\frac{z}{w}$ in modulus-argument form. [4 marks]

Answer .....

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EXAMPLE 5 · I DO

$z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ and $w = 3\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$. Find zw and $\frac{z}{w}$, with arguments in $(-\pi, \pi]$.

Hint Adding the arguments gives more than π. Take 2π off.

YOUR TURN · WE DO

B5 [5 marks]

$z = 4\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$ and $w = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$. Find zw and $\frac{z}{w}$ in modulus-argument form, with arguments in $(-\pi, \pi]$. Write $\frac{z}{w}$ in the form $a + bi$.

Answer .....

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C5 $z = 6\left(\cos\dfrac{3\pi}{8} + i\sin\dfrac{3\pi}{8}\right)$ and $w = 3\left(\cos\dfrac{\pi}{8} + i\sin\dfrac{\pi}{8}\right)$. Find zw and $\dfrac{z}{w}$, each in the form $a + bi$, exactly. [4 marks]

Answer .....

C6 Alex is asked for zw in modulus-argument form, with the argument in $(-\pi, \pi]$, where $z = 3(\cos 2.5 + i\sin 2.5)$ and $w = 2(\cos 1.9 + i\sin 1.9)$. He writes $zw = 6(\cos 4.4 + i\sin 4.4)$. What has Alex missed? Give the correct answer, and write zw in the form $a + bi$ to 3 significant figures. [5 marks]

Answer .....

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C7 $z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $w = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$. [5 marks]

Find $z + w$ in modulus-argument form, exactly.

Answer .....

D Exam-style questions

OCR exam style

D1

> Alex says that $\arg(z_1 + z_2) = \arg z_1 + \arg z_2 = \frac{2\pi}{3}$.

$z_1 = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ and $z_2 = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$.

(a) Find $z_1 + z_2$ in the form $a + bi$, and find $|z_1 + z_2|$ and $\arg(z_1 + z_2)$ exactly. [3 marks]

Answer .....

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(b) Show that Alex is wrong. Use the shape with vertices O, z_1, $z_1 + z_2$ and z_2 to explain why $\arg(z_1 + z_2)$ is halfway between $\arg z_1$ and $\arg z_2$ here. [3 marks]

(c) Find $z_1 z_2$ in the form $a + bi$, exactly. [3 marks]

Answer .....

Total for D1: 9 marks

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D2 In this question you must show detailed reasoning. $z = 2\left(\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right)$ and $w = 1 + i$.

(a) Write w in modulus-argument form. [2 marks]

Answer .....

(b) Find zw in modulus-argument form, and hence in the form $a + bi$, exactly. [4 marks]

Answer .....

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(c) Hence find the exact values of $\cos \frac{7\pi}{12}$ and $\sin \frac{7\pi}{12}$. [3 marks]

Answer .....

Total for D2: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $z = \cos\theta + i\sin\theta$, where $0 < \theta < \pi$. Show that $\dfrac{1+z}{1-z} = i\cot\dfrac{\theta}{2}$ [9 marks]

, by first writing $1+z$ and $1-z$ in modulus-argument form.

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Use $\cos\theta = 2\cos^2\frac{\theta}{2} - 1 = 1 - 2\sin^2\frac{\theta}{2}$ and $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$.

Hint 2 $\sin\alpha - i\cos\alpha = \cos\left(\alpha - \frac{\pi}{2}\right) + i\sin\left(\alpha - \frac{\pi}{2}\right)$.

Hint 3 A modulus must be positive. Check the factor in front is positive for $0 < \theta < \pi$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can change between $a + bi$ and $r(\cos\theta + i\sin\theta)$, exactly or to 3 significant figures. ○ ○ ○
- I can multiply and divide complex numbers by multiplying or dividing the moduli and adding or subtracting the arguments. ○ ○ ○
- I can bring an argument back into $(-\pi, \pi]$ or $[0, 2\pi)$. ○ ○ ○
- I can use a product found two ways to find exact values of sine and cosine. ○ ○ ○

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