Regions and set notation

92 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-regions/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Find the cartesian equation of the locus $|z - 3 + i| = 2$.

Answer .....

A2 Last lesson [2 marks]

Find the cartesian equation of the locus $|z| = |z - 6i|$.

Answer .....

A3 From A-Level Maths [2 marks]

Find the exact area of a sector of a circle with radius 6 and angle $\frac{\pi}{3}$.

Answer .....

A4 From A-Level Maths [2 marks]

Is the point $(3, 5)$ above or below the line $y = 2x + 1$?

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

PART 1 Regions from one condition

EXAMPLE 1 · I DO

Sketch the region $|z - 2 - i| \leq 3$ on an Argand diagram. Is the point $4 + 3i$ in the region?

Hint It is the circle $|z - 2 - i| = 3$ and every point inside it.

YOUR TURN · WE DO

B1 Sketch the region $|z + 1 - 2i| < 4$. Is the point $2 - i$ in the region? Show your reasoning. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 2 · I DO

Sketch the region $|z| \leq |z - 4|$.

Hint The edge is the perpendicular bisector of 0 and 4. The region is the side nearer to 0.

YOUR TURN · WE DO

B2 Find the cartesian equation of the edge of the region $|z - 2i| > |z + 4|$. Sketch the region, showing which side is shaded. [4 marks]

Answer .....

PRACTICE · YOU DO

C1 Sketch the region $|z - 4 + 2i| > 3$. Is the point $2 + i$ in the region? Show your reasoning. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C2 Alex sketches the region $|z| \leq |z - 6|$ as $x \leq 6$. Explain Alex's mistake, sketch the correct region, and write it in set notation in two ways. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

PART 2 Regions from arguments and two conditions

EXAMPLE 3 · I DO

Sketch the region $0 \leq \arg(z-1) \leq \frac{\pi}{3}$.

Hint Draw the half-lines $\arg(z-1)=0$ and $\arg(z-1)=\frac{\pi}{3}$ from the point 1.

YOUR TURN · WE DO

B3 Sketch the region $\frac{\pi}{4} < \arg(z+2i) < \frac{3\pi}{4}$. Show that the point $1+i$ is in the region. [4 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

EXAMPLE 4 · I DO

Sketch the region where $|z - 3| \leq 3$ and $\mathrm{Im}(z) \geq 1$. Find the corners of the region.

Hint Shade the part of the disc that is on or above the line $y = 1$.

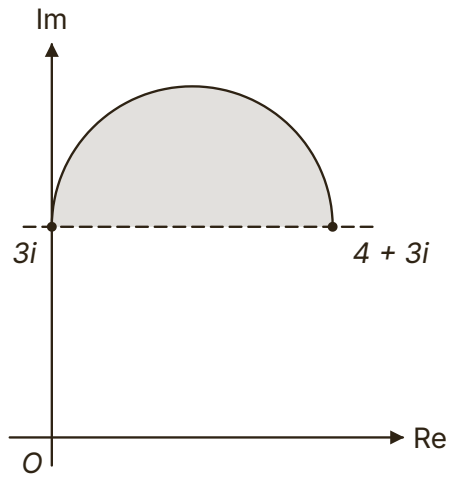
YOUR TURN · WE DO

B4 Sketch the region $\{z : |z| \leq 4\} \cap \{z : |z - 4| \leq |z|\}$. Find the corners of the region. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C3 The diagram shows a region R, shaded. The edge on the circle is included; the edge on the dashed line is not. The circle passes through $3i$ and $4 + 3i$, which are the ends of a diameter. Write R in set notation. [4 marks]



Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C4 Sketch the region $\left\{z : -\frac{\pi}{4} \leq \arg(z-2) \leq \frac{\pi}{2}\right\} \cap \{z : |z-2| \leq 3\}$, and find its exact area. [5 marks]

Answer .....

C5 $R = \{z : 2 \leq |z - 1 - i| \leq 4\}$. Sketch R. For each point, decide whether it is in R, showing your reasoning: (i) $3 + 4i$ (ii) $1 + 2i$ (iii) $5 - 2i$. Find the exact area of R. [6 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

PART 3 Greatest and least argument in a region

EXAMPLE 5 · I DO

z lies in the region $|z - 4 - 4i| \leq 2\sqrt{2}$. Find the greatest value of $\arg z$, and the point where it happens.

Hint The greatest argument is on the tangent from O, which is at right angles to the radius.

YOUR TURN · WE DO

B5 z lies in the region $|z - 2\sqrt{3} - 2i| \leq 2$. Find the greatest value of $\arg z$, and the point where it happens, exactly. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C6 In this question you must show detailed reasoning. The region R is $\{z : |z - 9 - 12i| \leq 5\}$. Find the greatest value of $\arg z$ for z in R, and the point where it happens. Give your answers to 3 significant figures. [5 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

C7 z lies in the region $|z - 3 - 3\sqrt{3}i| \leq 3$. Find the least and greatest values of $\arg z$, and the points where they happen, exactly. [6 marks]

Answer .....

D Exam-style questions

OCR exam style

D1 Sam writes $C = \{z : |z - 3| \leq 2\}$.

C is the set of points strictly inside the circle with centre 3 and radius 2.
$H = \left\{z : 0 < \arg(z - 1) < \frac{\pi}{3}\right\}$.

(a) Explain Sam's mistake, write C correctly in set notation, and say how its edge should be drawn. [2 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

(b) Show that the point $4 + 1.5i$ is in $C \cap H$. [4 marks]

(c) Find the exact area of $C \cap H$. [5 marks]

Answer .....

Total for D1: 11 marks

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

D2 In this question you must show detailed reasoning. The roots of $z^2 - 6z + 25 = 0$ are α and β. The region R is $\{z : |z - \alpha| \leq 5\} \cap \{z : |z - \beta| \leq 5\}$. Find the area of R, to 3 significant figures. [8 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 z lies in the region $\{z : |z - 3i| \leq 3\}$. Find the set of values of $\arg(z - 4)$, giving your answer exactly. Find the points where the least and greatest values happen. [9 marks]

Answer .....

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.

Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\arg(z-4)$ is the angle of the line from the point 4 to z. Sketch the disc and the point 4.

Hint 2 The smallest and largest angles are on the two tangents from 4 to the circle.

Hint 3 One tangent is the real axis. For the other, use the right-angled triangle with the radius, the tangent and the line from 4 to the centre.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch and shade a region, with solid edges for $\leq$ and dashed edges for $<$. ○ ○ ○

I can describe a shaded region in set notation, using $\cap$ for two conditions. ○ ○ ○

I can test whether a point is in a region by checking every condition. ○ ○ ○

I can find the least and greatest $|z|$ and $\arg z$ in a region, and areas of regions. ○ ○ ○

© 2026 Mathswariz. All rights reserved. Free for personal and classroom use. Not to be sold or republished.