# Roots of quadratic and cubic equations

91 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-roots-quad-cubic/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

Expand $(x-a)(x-b)(x-c)$.

Answer .....

**A2** From A-Level Maths [2 marks]

Write $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ as a single fraction.

Answer .....

**A3** From chapter 1 [2 marks]

Solve $z^2-6z+10=0$.

Answer .....

**A4** From A-Level Maths [2 marks]

Factorise $x^3-7x-6$ fully.

Answer .....

## B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).



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## PART 1 The roots of a quadratic

**EXAMPLE 1 · I DO**

The roots of $2x^2 - 6x + 3 = 0$ are $\alpha$ and $\beta$. Find $\alpha + \beta$, $\alpha\beta$ and $\alpha^2 + \beta^2$.

**Hint** Divide by 2 first: $x^2 - 3x + \frac{3}{2} = 0$. Then $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

**YOUR TURN · WE DO**

**B1** The roots of $3x^2 + 4x - 2 = 0$ are $\alpha$ and $\beta$. Find $\alpha^2 + \beta^2$ and $\frac{1}{\alpha} + \frac{1}{\beta}$. [4 marks]

Answer .....


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**EXAMPLE 2 · I DO**

The roots of $x^2 - 5x + 2 = 0$ are $\alpha$ and $\beta$. Find $\alpha^3 + \beta^3$ and $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$.

**Hint** Expand $(\alpha + \beta)^3$ and take away the terms you do not want.

**YOUR TURN · WE DO**

**B2** The roots of $2x^2 + x - 4 = 0$ are $\alpha$ and $\beta$. Find $\alpha^3 + \beta^3$ and $(\alpha - \beta)^2$. [4 marks]

Answer .....

**PRACTICE · YOU DO**

**C1** The roots of $x^2 + 3x - 6 = 0$ are $\alpha$ and $\beta$. Find $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$. [3 marks]

Answer .....


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**C2** One root of $x^2 - kx + 12 = 0$ is three times the other root. Find the possible values of $k$. [4 marks]

Answer .....


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## PART 2 The roots of a cubic

**EXAMPLE 3 · I DO**

The roots of $2x^3 - 4x^2 + 3x - 5 = 0$ are $\alpha$, $\beta$ and $\gamma$. Find $\sum \alpha$, $\sum \alpha\beta$, $\alpha\beta\gamma$, $\sum \alpha^2$ and $\sum \frac{1}{\alpha}$.

**Hint** For $ax^3 + bx^2 + cx + d = 0$: $\sum \alpha = -\frac{b}{a}$, $\sum \alpha\beta = \frac{c}{a}$, $\alpha\beta\gamma = -\frac{d}{a}$. The signs alternate.

**YOUR TURN · WE DO**

**B3** The roots of $x^3 + 3x^2 - 2x + 4 = 0$ are $\alpha$, $\beta$ and $\gamma$. Find $\sum \alpha^2$ and $\sum \frac{1}{\alpha}$. [5 marks]

Answer .....


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**EXAMPLE 4 · I DO**

The roots of $x^3 - 9x^2 + 23x + k = 0$ form an arithmetic sequence. Find the roots and the value of $k$.

**Hint** Call the roots $a - d$, $a$ and $a + d$. Their sum tells you $a$.

**YOUR TURN · WE DO**

**B4** The roots of $x^3 + 6x^2 + 3x + k = 0$ form an arithmetic sequence. Find the value of $k$ and the three roots. [5 marks]

Answer .....


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**C3**

> Sam writes: "$\sum \alpha = \frac{4}{5}$, $\sum \alpha\beta = -\frac{2}{5}$ and $\alpha\beta\gamma = 2$."

The roots of $5x^3 + 4x^2 - 2x + 10 = 0$ are $\alpha$, $\beta$ and $\gamma$.

**(a)** Two of Sam's values are wrong. Write down the correct values. [2 marks]

Answer .....

**(b)** Find $\sum \alpha^2$. [2 marks]

Answer .....

**Total for C3: 4 marks**



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**C4** The roots of $x^3 - 2x^2 + 5x - 3 = 0$ are $\alpha$, $\beta$ and $\gamma$. Find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [4 marks]

Answer .....

**C5** The roots of $x^3 + 4x^2 - x + 6 = 0$ are $\alpha$, $\beta$ and $\gamma$. Find $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. [5 marks]

Answer .....


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## PART 3 Complex roots and new equations

**EXAMPLE 5 · I DO**

$2 + i$ is a root of $x^3 + px^2 + qx + 15 = 0$, where $p$ and $q$ are real. Find the other roots, $p$ and $q$.

**Hint** $2 - i$ is also a root. The product of all three roots is $-15$, which gives the third root at once.

**YOUR TURN · WE DO**

**B5** $1 - 3i$ is a root of $x^3 + px^2 + qx + 30 = 0$, where $p$ and $q$ are real. Find the other roots, $p$ and $q$. [5 marks]

Answer .....


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**EXAMPLE 6 · I DO**

The roots of $x^2 - 4x + 7 = 0$ are $\alpha$ and $\beta$. Find a quadratic equation with roots $\alpha^2$ and $\beta^2$.

**Hint** A quadratic with roots $p$ and $q$ is $x^2 - (p+q)x + pq = 0$. Find the sum and product of the new roots.

**YOUR TURN · WE DO**

**B6** The roots of $2x^2 + 3x - 1 = 0$ are $\alpha$ and $\beta$. Find a quadratic equation, with integer coefficients, whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [5 marks]

Answer .....


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**C6** Find a cubic equation, with integer coefficients, whose roots are $\frac{1}{2}$, $-3$ and $\frac{2}{3}$. [4 marks]

Answer .....

**C7** The roots of $3x^2 - 5x + 4 = 0$ are $\alpha$ and $\beta$. Find a quadratic equation, with integer coefficients, whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [5 marks]

Answer .....


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**D1**

> Kai writes: "$\alpha^2 + \beta^2 + \gamma^2$ is positive, so all three roots are real."

The roots of $x^3 - 2x^2 - 3x + 10 = 0$ are $\alpha$, $\beta$ and $\gamma$.

**(a)** Find $\alpha^2 + \beta^2 + \gamma^2$. [2 marks]

Answer .....

**(b)** Show that $x = -2$ is a root, and find the other two roots. [4 marks]

Answer .....


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**(c)** Explain what is wrong with Kai's reasoning. [2 marks]

**Total for D1: 8 marks**

**D2** The roots of $x^3 - 14x^2 + kx - 64 = 0$ form a geometric sequence.

**(a)** Write the roots as $\frac{a}{r}$, $a$ and $ar$. Show that $a = 4$. [2 marks]

**(b)** Find the three roots. [4 marks]

Answer .....


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**(c)** Find the value of $k$. [2 marks]

Answer .....

**(d)** Hence find the sum of the squares of the roots, using the coefficients, and check it using the roots. [2 marks]

Answer .....

**Total for D2: 10 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** The roots of $x^3 - 3x + 1 = 0$ are $\alpha$, $\beta$ and $\gamma$. Find $\alpha^3 + \beta^3 + \gamma^3$ and $\alpha^4 + \beta^4 + \gamma^4$. [8 marks]

Answer .....



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** Write down the three symmetric functions. Then find the sum of the squares.

**Hint 2** Each root satisfies the equation, so $\alpha^3 = 3\alpha - 1$. Add this up for all three roots.

**Hint 3** Multiply the equation by $x$ to get a fourth power in terms of lower powers.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can write down $\sum \alpha$, $\sum \alpha\beta$ and $\alpha\beta\gamma$ for a cubic, with the right signs. | ○ ○ ○ |
| I can find sums of squares, cubes and reciprocals of the roots from the coefficients. | ○ ○ ○ |
| I can use a condition on the roots, such as an arithmetic sequence or a complex root, to find the roots and coefficients. | ○ ○ ○ |
| I can form a new equation with integer coefficients whose roots are given in terms of the old roots. | ○ ○ ○ |



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