The scalar product and angles between lines

90 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-scalar-product/

Date

A Do Now

Quick questions to start.

A1 From A-Level Maths [2 marks]

Find the magnitude of $\begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$.

Answer .....

A2 Needed today [1 mark]

Find $\cos^{-1}(-0.4)$ in degrees, to 1 decimal place.

Answer .....

A3 Last lesson [2 marks]

Write down a direction vector of the line $\frac{x-1}{3} = \frac{2-y}{4} = \frac{z}{2}$.

Answer .....

A4 Needed today [2 marks]

A is $(2, -1, 5)$ and B is $(4, 3, 1)$. Find $\overrightarrow{AB}$ and the length AB.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 The scalar product and perpendicular vectors

EXAMPLE 1 · I DO

$\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$. Find $\mathbf{a} \cdot \mathbf{b}$. Is $\mathbf{a}$ perpendicular to $\mathbf{b}$? Show that $\mathbf{a}$ is perpendicular to $\begin{pmatrix} 2 \\ 7 \\ 2 \end{pmatrix}$.

Hint Multiply matching components and add. The answer is a number, not a vector.

YOUR TURN · WE DO

B1 [3 marks]

$\mathbf{p} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ 1 \\ -5 \end{pmatrix}$. Show that $\mathbf{p}$ and $\mathbf{q}$ are perpendicular, and find $\mathbf{p} \cdot \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix}$.

Answer .....

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EXAMPLE 2 · I DO

$\mathbf{a} = \begin{pmatrix} 2 \\ p \\ 8 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ p \end{pmatrix}$. Find the value of p for which $\mathbf{a}$ and $\mathbf{b}$ are parallel, and the value for which they are perpendicular.

Hint Parallel means $\mathbf{a} = k\mathbf{b}$ for one number k. Perpendicular means $\mathbf{a} \cdot \mathbf{b} = 0$.

YOUR TURN · WE DO

B2 [5 marks]

$\mathbf{a} = \begin{pmatrix} 2 \\ p \\ 12 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}$. Find the value of p for which $\mathbf{a}$ and $\mathbf{b}$ are parallel, and the value for which they are perpendicular.

Answer .....

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C1 [3 marks]

$\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ -5 \\ 1 \end{pmatrix}$. Show that $\mathbf{u}$ and $\mathbf{v}$ are perpendicular, and find $\mathbf{u} \cdot \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$.

Answer .....

C2 [5 marks]

$\mathbf{a} = \begin{pmatrix} 2 \\ k \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} k \\ 3 \\ -5 \end{pmatrix}$. Find the value of k for which $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. Show that there is no value of k for which they are parallel.

Answer .....

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PART 2 Angles between vectors and between lines

EXAMPLE 3 · I DO

Find the angle between $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -4 \\ 0 \\ 3 \end{pmatrix}$.

Hint Use $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$. A negative cosine gives an obtuse angle, and that is the angle between the vectors.

YOUR TURN · WE DO

B3 Find the angle between $\begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$. [4 marks]

Answer .....

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EXAMPLE 4 · I DO

Find the acute angle between the lines $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

Hint Use the two direction vectors, not the points. The angle between lines is the acute one.

YOUR TURN · WE DO

B4 Find the acute angle between the lines $\frac{x-1}{2} = \frac{y+3}{-2} = z$ and $\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 1 \\ -2 \end{pmatrix}$. [4 marks]

Answer .....

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C3 [4 marks]

Find the acute angle between the line $\mathbf{r} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$ and the y-axis.

Answer .....

C4 [5 marks]

$\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$. Sam writes: "$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} 3 \\ -8 \\ -10 \end{pmatrix}$."

Explain Sam's mistake, then find the angle between $\mathbf{a}$ and $\mathbf{b}$.

Answer .....

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C5 A, B and C are $(3, 1, 2)$, $(1, 2, 4)$ and $(2, 4, 4)$. Show that angle ABC is a right angle, and find angle BAC. [6 marks]

Answer .....

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PART 3 The foot of a perpendicular

EXAMPLE 5 · I DO

L is the line $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and P is $(6, 4, -2)$. Find the foot of the perpendicular from P to L, and the distance from P to L.

Hint Let F be a general point on L. Then $\overrightarrow{PF}$ is perpendicular to the direction of L, so their scalar product is 0.

YOUR TURN · WE DO

B5 L is the line $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ and P is $(6, 2, -2)$. Find the coordinates of the foot of the perpendicular from P to L, and the exact distance from P to L. [6 marks]

Answer .....

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C6 [4 marks]

L is the line $\mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$. C is the point on L nearest the origin O. Find the coordinates of C and the exact distance OC.

Answer .....

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C7 In this question you must show detailed reasoning. The line L has equation $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and P is the point $(5, 3, 4)$. The point P' is the reflection of P in L. Find the coordinates of P', and the exact distance from P to L. [7 marks]

Answer .....

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D1

> Alex writes: "Two vectors are parallel when their scalar product is 1, so I solve $\mathbf{a} \cdot \mathbf{b} = 1$."

$\mathbf{a} = \begin{pmatrix} p \\ 2 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} p+3 \\ -4 \\ 2 \end{pmatrix}$, where p is a constant.

(a) Find the values of p for which $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. [3 marks]

Answer .....

(b) Explain why Alex's method is wrong. [2 marks]

Answer .....

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(c) Find the value of p for which $\mathbf{a}$ and $\mathbf{b}$ are parallel, and show that $\mathbf{a} \cdot \mathbf{b}$ is then equal to $-|\mathbf{a}||\mathbf{b}|$. [4 marks]

Total for D1: 9 marks

D2 The points A and B are $(2, -1, 3)$ and $(4, 3, -1)$. The point C lies on the line $\mathbf{r} = (1, 1, 0) + t(0, 1, 1)$, and on the circle with diameter AB.

(a) Explain why $\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$. [2 marks]

Answer .....

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(b) Find the two possible positions of C. Give your answers as coordinates. [5 marks]

Answer .....

(c) In this question you must show detailed reasoning. Hence, for the position of C with positive z-coordinate, find the exact size of angle CAB. [3 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A is $(1, 2, 3)$ and B is $(3, 0, 5)$. The point C lies on the line $\mathbf{r} = (0, 1, 0) + t(1, 0, 1)$. Find all the positions of C for which triangle ABC has a right angle. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write C in terms of t. The right angle could be at A, at B or at C: three cases.

Hint 2 At A you need $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0$. Do the same at B.

Hint 3 At C you need $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$, which gives a quadratic in t.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out a scalar product and use it to show two vectors are perpendicular.	○ ○ ○
I can find the angle between two vectors, and the acute angle between two lines from their directions.	○ ○ ○
I can find a constant that makes two vectors parallel or perpendicular.	○ ○ ○
I can find the foot of the perpendicular from a point to a line, and use $\mathbf{a} \cdot \mathbf{b} = 0$ for right angles.	○ ○ ○

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