# Transformations in two dimensions

97 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-transform-2d/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

$M = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$. Work out $M \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

Answer .....

**A2** Needed today [2 marks]

Write down the exact values of $\sin 60°$ and $\cos 135°$.

Answer .....

**A3** From chapter 3 [2 marks]

Find the inverse of $\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$.

Answer .....

**A4** From A-Level Maths [1 mark]

The point $(3, -1)$ is reflected in the line $y = x$. Write down its image.

Answer .....

## B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).



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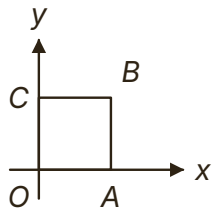
## PART 1 Images and the columns of a matrix

**EXAMPLE 1 · I DO**

$M = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$. Find the images of the corners $O$, $A(1, 0)$, $B(1, 1)$ and $C(0, 1)$ of the unit square.




**Hint** Multiply $M$ by each corner written as a column. Notice where the columns of $M$ come from.

**YOUR TURN · WE DO**

**B1** $M = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$. The triangle $PQR$ has $P(1, 2)$, $Q(3, 0)$ and $R(-1, 1)$. Find the images of $P$, $Q$ and $R$ . [4 marks]

Answer .....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**EXAMPLE 2 · I DO**

Write down the matrix for (i) a reflection in the line $y = -x$ (ii) a rotation of $90^\circ$ clockwise about the origin.

**Hint** Ask where $(1, 0)$ goes and where $(0, 1)$ goes. Those two images are the two columns.

**YOUR TURN · WE DO**

**B2** Write down the matrix for (i) a rotation of $90^\circ$ anticlockwise about the origin (ii) a reflection in the $y$-axis (iii) a reflection in the line $y = x$ (iv) an enlargement with scale factor $-2$, centre the origin. [4 marks]

Answer .....


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**C1** $M = \begin{pmatrix} a & 2 \\ b & -1 \end{pmatrix}$ maps the point $(1, 3)$ to $(9, 1)$. Find $a$ and $b$, and find the image of $(2, -1)$. [4 marks]

Answer .....

**C2** Sam writes the matrix for a rotation of $90°$ clockwise about the origin as $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Explain Sam's mistake, write down the correct matrix, and find the image of $(3, 5)$ under the rotation. [4 marks]

Answer .....


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**C3** The matrix $\mathbf{M}$ maps the point $(1, 0)$ to $(3, 1)$ and the point $(1, 2)$ to $(1, 5)$. Find $\mathbf{M}$. Find the point whose image under $\mathbf{M}$ is $(5, 4)$. [6 marks]

Answer .....


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## PART 2 Rotations, stretches and shears

**EXAMPLE 3 · I DO**

Find, using exact values, the matrix for a rotation of $120^\circ$ anticlockwise about the origin. Find the image of the point $(4, 2)$.

**Hint** An anticlockwise rotation through $\theta$ has matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$.

**YOUR TURN · WE DO**

**B3** Find, using exact values, the matrix for a rotation of $45^\circ$ clockwise about the origin. Find the image of the point $(6, 2)$. [5 marks]

Answer .....


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**EXAMPLE 4 · I DO**

Find the matrix for (i) a stretch parallel to the $x$-axis, scale factor $3$, with the $y$-axis invariant (ii) a shear with the $x$-axis invariant that maps $(1, 2)$ to $(7, 2)$.

**Hint** A shear with the $x$-axis invariant has matrix $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$. Use the given point to find $k$.

**YOUR TURN · WE DO**

**B4** Find the matrix for (i) a stretch parallel to the $y$-axis, scale factor $\frac{1}{2}$, with the $x$-axis invariant (ii) a shear with the $y$-axis invariant that maps $(2, 1)$ to $(2, -5)$. [5 marks]

Answer .....


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**C4** A shear with the $x$-axis invariant maps $(3, 2)$ to $(-1, 2)$. Find the matrix of the shear. Find the point whose image under the shear is $(5, 4)$. [5 marks]

Answer .....

**C5** $S$ is the stretch parallel to the $y$-axis, scale factor $k$, with the $x$-axis invariant, where $k > 0$. $H$ is the shear with the $y$-axis invariant that maps $(1, 0)$ to $(1, 2)$.

**(a)** Write down the matrices of $S$ and $H$. [2 marks]

Answer .....



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**(b)** The triangle with vertices $(0, 0)$, $(4, 0)$ and $(4, 3)$ is transformed by $S$. The longest side of the image has length $4\sqrt{5}$. Find $k$. [4 marks]

Answer .....

**(c)** Find the image of $(3, -1)$ under $H$, and the point whose image under $H$ is $(2, 7)$. [3 marks]

Answer .....

**Total for C5: 9 marks**


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## PART 3 Describing transformations

**EXAMPLE 5 · I DO**

Describe fully the transformation represented by (i) $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ (ii) $\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$.

**Hint** Name the transformation, then give every detail: an angle and a direction, a centre, a scale factor.

**YOUR TURN · WE DO**

**B5** Describe fully the transformation represented by (i) $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$. [6 marks]

Answer .....


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**C6** Describe fully the transformations represented by $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$ and by $\begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$. [4 marks]

Answer .....



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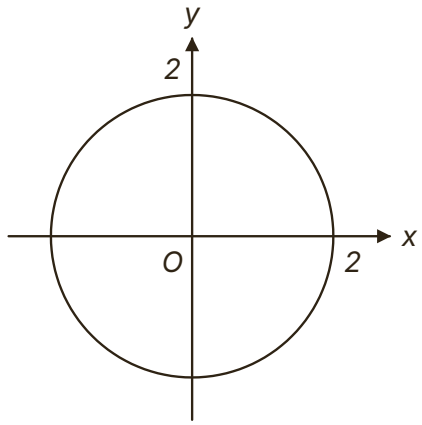
**C7** The circle $x^2 + y^2 = 4$ is transformed by $S$, the stretch parallel to the $y$-axis, scale factor $3$, with the $x$-axis invariant.




**(a)** Write down the matrix of $S$, and the images of $(2, 0)$ and $(0, 2)$ [2 marks]

.

Answer .....

**(b)** Show that the image of the circle has equation $9x^2 + y^2 = 36$. [4 marks]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**(c)** Find the exact coordinates of the points where the image meets the line $y = 3x$. [3 marks]

Answer .....

**Total for C7: 9 marks**

## D Exam-style questions

OCR exam style

**D1** Alex says: "$S$ represents a stretch parallel to the $x$-axis, scale factor $3$."

The matrix $S = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$.

**(a)** By finding the images of $(1, 0)$ and $(0, 1)$, explain why Alex is wrong. [2 marks]



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**(b)** Describe fully the transformation $S$ represents. [2 marks]

Answer .....

**(c)** The triangle with vertices $(0, 0)$, $(2, 0)$ and $(1, 2)$ is transformed by $S$. Find the vertices of the image, and show that the triangle and its image have the same area. [3 marks]

Answer .....

**Total for D1: 7 marks**



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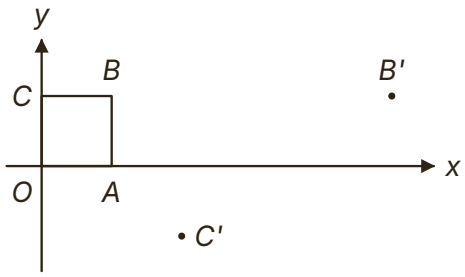
**D2** The unit square $OABC$ has $A(1, 0)$, $B(1, 1)$ and $C(0, 1)$. The transformation $T$, with matrix $M$, maps $B$ to $B'(5, 1)$ and $C$ to $C'(2, -1)$.

**(a)** Find $M$. [3 marks]




Answer .....

**(b)** Find the point whose image under $T$ is $(12, 1)$. [3 marks]

Answer .....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**(c)** Find the equation of the image of the line $y = x + 1$ under $T$. [4 marks]

Answer .....

**Total for D2: 10 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** A rotation about the origin maps the point $(7, 1)$ to a point on the line $y = x$. Find the two possible matrices for the rotation, using exact values. [8 marks]

Answer .....



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** A rotation about the origin does not change a point's distance from the origin. Which points on $y = x$ are that far away?

**Hint 2** Write the rotation as $\begin{pmatrix} c & -s \\ s & c \end{pmatrix}$, where $c = \cos\theta$ and $s = \sin\theta$, and multiply it by $(7, 1)$.

**Hint 3** Solve the two equations for $c$ and $s$. Do it once for each possible image.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can find the matrix of a transformation from the images of $(1, 0)$ and $(0, 1)$. | ○ ○ ○ |
| I can write down matrices for reflections, rotations, enlargements, stretches and shears. | ○ ○ ○ |
| I can describe fully the transformation that a matrix represents. | ○ ○ ○ |
| I can find the image of a point, a shape, a line or a curve. | ○ ○ ○ |


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