Successive transformations and the determinant

99 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-transform-successive/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Write down the matrix for (i) a rotation of $90°$ anticlockwise about the origin (ii) a reflection in the line $y = x$.

Answer .....

A2 Needed today [2 marks]

Work out $\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 4 & 1 \end{pmatrix}$.

Answer .....

A3 From chapter 3 [3 marks]

Find the determinant and the inverse of $\begin{pmatrix} 4 & 3 \\ 2 & 2 \end{pmatrix}$.

Answer .....

A4 From A-Level Maths [2 marks]

Find the area of the triangle with vertices $(0, 0)$, $(4, 0)$ and $(1, 3)$.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 One transformation followed by another

EXAMPLE 1 · I DO

The transformation T is a reflection in the x-axis followed by a rotation of 90° anticlockwise about the origin. Find the matrix of T and describe T as a single transformation.

Hint If X is done first and R second, a point $\mathbf{p}$ goes to $R(X\mathbf{p})$. So the single matrix is RX: the first one goes on the right.

YOUR TURN · WE DO

B1 The transformation T is a rotation of 90° anticlockwise about the origin followed by a reflection in the x-axis. Find the matrix of T and describe T as a single transformation. [4 marks]

Answer .....

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C1 $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Sam says that the matrix for "A followed by B" is $AB = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$. Explain Sam's mistake and find the correct matrix. [4 marks]

Answer .....

C2 $M = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. By considering the transformation that M represents, explain why $M^2 = I$. Then verify this by multiplying. [4 marks]

Answer .....

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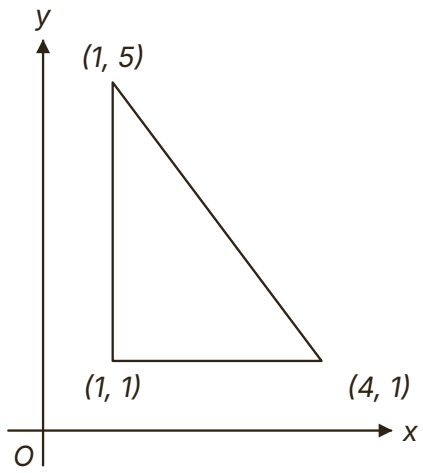
C3 The triangle with vertices $(1, 1)$, $(4, 1)$ and $(1, 5)$ is transformed by a stretch parallel to the x-axis, scale factor 3, with the y-axis invariant, followed by a reflection in the line $y = -x$.



(a) Find the single matrix for the combined transformation. [3 marks]

Answer .....

(b) Find the area of the image, and say whether the orientation is kept or reversed. [3 marks]

Answer .....

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(c) Find the image of the vertex $(4, 1)$. [2 marks]

Answer .....

Total for C3: 8 marks

PART 2 Area, orientation and the determinant

EXAMPLE 2 · I DO

A shape of area 5 is transformed by $M = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$. Find the area of the image and say whether the orientation is kept or reversed.

Hint The determinant is the area scale factor. A negative determinant means the orientation is reversed, and the area is still positive.

YOUR TURN · WE DO

B2 A square of side 3 is transformed by $M = \begin{pmatrix} 2 & 5 \\ 1 & -3 \end{pmatrix}$. Find the area of the image and say whether the orientation is kept or reversed. [4 marks]

Answer .....

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EXAMPLE 3 · I DO

A shape of area 2 is transformed by $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ and then by $B = \begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$. Find the area of the final image.

Hint You do not need to multiply the matrices. $\det(BA) = \det B \times \det A$.

YOUR TURN · WE DO

B3 A shape of area 3 is transformed by $P = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and then by $Q = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}$. Find the area of the final image and say whether the orientation is kept or reversed. [5 marks]

Answer .....

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C4 A triangle of area 6 is transformed by $\begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix}$. Alex says the image has area -30. Explain what is wrong, and give the area and the effect on orientation. [4 marks]

Answer .....

C5 $M = \begin{pmatrix} k & 4 \\ k+1 & k+2 \end{pmatrix}$.

(a) Find the exact values of k for which M is singular. [3 marks]

Answer .....

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(b) Given $k = 2$, find the area of the image of the triangle with vertices $(0, 0)$, $(3, 1)$ and $(1, 2)$ under M. [3 marks]

Answer .....

(c) Given $k = 1 + \sqrt{5}$, every point is mapped onto one line through the origin. Find its equation in the form $y = mx$, with m exact. [3 marks]

Answer .....

Total for C5: 9 marks

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PART 3 Undoing a transformation with the inverse

EXAMPLE 4 · I DO

$M = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$. The point P is mapped by M to $(7, 18)$. Find P.

Hint The inverse matrix represents the inverse transformation: it sends each image back to where it came from.

YOUR TURN · WE DO

B4 $M = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$. The point P is mapped by M to $(5, 1)$. Find P. [4 marks]

Answer .....

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EXAMPLE 5 · I DO

The transformation M followed by a rotation R of 90° anticlockwise about the origin has matrix $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$. Find M.

Hint The combined matrix is RM. To remove R, multiply by R^{-1} on the LEFT, because R is on the left.

YOUR TURN · WE DO

B5 A reflection in the x-axis followed by the transformation N has matrix $\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$. Find N. [5 marks]

Answer .....

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C6 $M = \begin{pmatrix} 4 & 3 \\ 1 & 1 \end{pmatrix}$. The transformation M followed by a reflection in the y-axis maps the point Q to $(-6, 1)$. Find Q. [5 marks]

Answer .....

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C7 In this question you must show detailed reasoning. The transformation T has matrix $\begin{pmatrix} a & 2 \\ 3 & a+1 \end{pmatrix}$, where $a > 0$. T maps a shape of area 4 to a shape of area 24, and keeps its orientation. Find the point whose image under T is $(7, 17)$. [7 marks]

Answer .....

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D Exam-style questions

OCR exam style

D1

> Sam says: "$\det C = 2$ is positive, so T keeps the orientation of a shape."

$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$. The transformation T is A, followed by B, followed by C.

(a) Show that Sam is wrong. [3 marks]

(b) A shape of area 5 is transformed by T. Find the area of its image. [2 marks]

Answer .....

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(c) Find the matrix of T. [3 marks]

Answer .....

Total for D1: 8 marks

D2 $A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$ represents a shear and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$. The transformation C is A followed by B.

(a) Find the matrix of C. [3 marks]

Answer .....

(b) Show that B followed by A is a different transformation. [2 marks]

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(c) Find C^{-1}, and show that $C^{-1} = A^{-1}B^{-1}$. Explain this in terms of the transformations. [3 marks]

Answer .....

(d) The point P is mapped by C to $(5, 2)$. Find P. [2 marks]

Answer .....

Total for D2: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The transformation T is a rotation about the origin followed by an enlargement, centre the origin, with a positive scale factor. T maps the point $(3, 4)$ to $(-10, 5)$. Find the matrix of T, the scale factor, the angle of the rotation, and the area of the image of the unit square. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 A rotation followed by an enlargement with centre the origin has a matrix of the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$.

Hint 2 Multiply that matrix by $(3, 4)$ and set the result equal to $(-10, 5)$. Solve the two equations for a and b.

Hint 3 The scale factor is how many times longer $(-10, 5)$ is than $(3, 4)$. The area scale factor is the determinant.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the matrix of one transformation followed by another, multiplying in the right order.	○ ○ ○
I can use the determinant as the area scale factor and say whether orientation is kept or reversed.	○ ○ ○
I can use $\det(AB) = \det A \det B$ and inverse matrices to undo a transformation.	○ ○ ○
I can explain what a singular matrix does to the plane.	○ ○ ○

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