Equations of lines in two and three dimensions

84 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-vector-lines/

Date

A Do Now

Quick questions to start.

A1 From A-Level Maths [2 marks]

Find the magnitude of $\begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix}$.

Answer .....

A2 Needed today [2 marks]

A is $(3, -1, 4)$ and B is $(1, 5, 2)$. Find the vector $\overrightarrow{AB}$.

Answer .....

A3 From A-Level Maths [2 marks]

Find the equation of the straight line through $(-1, 4)$ and $(3, -4)$ in the form $y = mx + c$.

Answer .....

A4 Needed today [2 marks]

Find the value of λ for which $(1 + 2\lambda,\ 4 - \lambda,\ 3\lambda) = (7,\ 1,\ 9)$, and check it fits all three.

Answer .....

B Examples and practice

A-Level Further

Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

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PART 1 A vector equation through two points

EXAMPLE 1 · I DO

Find a vector equation of the line through $A(1, 4, -2)$ and $B(3, 1, 5)$.

Hint The direction is $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$. Start the answer with $\mathbf{r} =$.

YOUR TURN · WE DO

B1 Find a vector equation of the line through $P(-2, 5, 1)$ and $Q(4, 3, -1)$. [3 marks]

Answer .....

PRACTICE · YOU DO

C1 Find a vector equation of the line through $C(0, -3, 2)$ and $D(5, 1, -4)$. [3 marks]

Answer .....

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C2 Sam writes: "The line through $A(2, 0, -1)$ and $B(5, 4, 1)$ is $\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 7 \\ 4 \\ 0 \end{pmatrix}$." Explain the two mistakes Sam has made, and write a correct equation of the line. [4 marks]

Answer .....

C3 A is $(1, 2, 3)$ and B is $(a, 8, b)$. The line through A and B is parallel to $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$. Find a and b. [4 marks]

Answer .....

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PART 2 Vector form and cartesian form

EXAMPLE 2 · I DO

In 2-D, write the line $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ in the form $ax + by = c$.

Then write $x - 3y = 6$ in vector form.

Hint Write x and y in terms of λ, then get rid of λ.

YOUR TURN · WE DO

B2 Write $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ in the form $ax + by = c$. Then write $2x - 5y = 7$ in vector form. [4 marks]

Answer .....

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EXAMPLE 3 · I DO

Write $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix}$ in cartesian form. Then write $\frac{x+1}{2} = \frac{y-4}{3} = \frac{z-2}{-6}$ in vector form.

Hint In cartesian form each fraction equals λ: $\frac{x-a_1}{u_1} = \frac{y-a_2}{u_2} = \frac{z-a_3}{u_3}$.

YOUR TURN · WE DO

B3 [4 marks]

Write $\mathbf{r} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}$ in cartesian form. Then write $\frac{x-5}{3} = \frac{y+2}{-1} = \frac{z}{6}$ in vector form.

Answer .....

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C4 [4 marks]

In 2-D, the line l_1 is $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and the line l_2 is $x + 4y = 7$. Write l_1 in the form $y = mx + c$, and find the coordinates of the point where l_1 and l_2 meet.

Answer .....

C5 [4 marks]

Write $\mathbf{r} = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -1 \\ 2 \end{pmatrix}$ in cartesian form. Then write $\dfrac{x-1}{2} = \dfrac{y+4}{-3} = \dfrac{z}{5}$ in vector form.

Answer .....

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C6 [4 marks]

Write the line $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix}$ in cartesian form.

Answer .....

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PART 3 Points on a line

EXAMPLE 4 · I DO

L is the line $\mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$.

Show that $A(5, 7, -2)$ lies on L and that $B(2, -5, 3)$ does not. Find the point on L where $z = 0$.

Hint Find λ from one component, then check that the same λ gives the other two.

YOUR TURN · WE DO

B4 L is the line $\mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$. [5 marks]

Show that $A(4, -3, 7)$ lies on L and that $B(1, -1, 7)$ does not. Find the point on L where $y = 0$.

Answer .....

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EXAMPLE 5 · I DO

L is the line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and B is $(3, 4, -1)$. Find the coordinates of the points on L whose distance from B is $5\sqrt{2}$.

Hint Write a general point P on L in terms of λ, then find $|\overrightarrow{BP}|^2$ and set it equal to 50.

YOUR TURN · WE DO

B5 [5 marks]

L is the line $\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and C is $(0, 3, 4)$. Find the coordinates of the points on L whose distance from C is $\sqrt{65}$.

Answer .....

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C7 [4 marks]

L is the line $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$. Show that $A(7, 1, 0)$ lies on L and that $B(5, 0, 2)$ does not.

Answer .....

C8 [5 marks]

L is the line $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and A is $(5, 1, 3)$. Find the coordinates of the points on L whose distance from A is $4\sqrt{2}$.

Answer .....

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D1

> Alex writes: "A direction vector of L is $\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}$."

The line L has cartesian equations $\dfrac{x-3}{2} = \dfrac{1-y}{4} = z+2$.

(a) Explain why Alex is wrong, and give a correct direction vector of L. [2 marks]

Answer .....

(b) Write L in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [2 marks]

Answer .....

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(c) Q is the point $(1, -3, 0)$. Find the coordinates of the two points on L whose distance from Q is $\sqrt{17}$. [5 marks]

Answer .....

Total for D1: 9 marks

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D2 In this question you must show detailed reasoning. The line L passes through $A(3, 0, -1)$ and $B(1, 4, 5)$. The line M has cartesian equations $\frac{x+1}{2} = \frac{y-8}{-4} = \frac{z-11}{-6}$. Show that L and M are the same line. [6 marks]

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The line M has equations $x = 4,\ z = 1$. Find the cartesian equations of the line that passes through $(2, 3, -1)$ and meets both the z-axis and M. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 A point on the z-axis is $(0, 0, q)$. Write the direction of the line from $(2, 3, -1)$ to it.

Hint 2 A point on the line is $(2, 3, -1) + s(-2, -3, q + 1)$. On M, $x = 4$, which fixes s.

Hint 3 Then $z = 1$ gives one equation for q.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the vector equation of a line through two points, starting with $\mathbf{r} =$. ○ ○ ○

I can convert a line between vector form and cartesian form in 2-D and 3-D. ○ ○ ○

I can test whether a point lies on a line by checking one value of λ fits every component. ○ ○ ○

I can find points on a line that meet a condition, such as a given distance from a point. ○ ○ ○

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