# The vector product

84 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/ocr-further-maths-vector-product/

Date

## A Do Now

Quick questions to start.

**A1** Last lesson [2 marks]

Find the point where the lines $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ meet.

Answer .....

**A2** From chapter 3 [1 mark]

Find the value of the determinant $\begin{vmatrix} 3 & 1 \\ -2 & 4 \end{vmatrix}$.

Answer .....

**A3** Needed today [2 marks]

Show that $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ is perpendicular to $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

**A4** Needed today [2 marks]

Write the line $\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ in Cartesian form.

Answer .....



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Your teacher works each example with you (I do). You try the one beside it together (We do). Then practise that part on your own (You do).

## PART 1 Working out a vector product

**EXAMPLE 1 · I DO**

Find $\mathbf{a} \times \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

**Hint** Write it as a determinant with $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ on the top row. The $\mathbf{j}$ term has a minus in front of it.

**YOUR TURN · WE DO**

**B1** Find $\mathbf{a} \times \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$. [3 marks]

Answer .....



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**C1** [4 marks]

$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$. Find $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$, and state how they are related. Write down $\mathbf{a} \times \mathbf{a}$, giving a reason.

Answer .....

**C2** [5 marks]

$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$. Sam writes: "$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 0 \\ -14 \\ 7 \end{pmatrix}$."

Explain Sam's mistake. Find $\mathbf{a} \times \mathbf{b}$, and show that your answer is perpendicular to $\mathbf{a}$ and to $\mathbf{b}$.

Answer .....



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## PART 2 Perpendicular and parallel vectors

**EXAMPLE 2 · I DO**

Find a vector perpendicular to both $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 0 \\ 2 \end{pmatrix}$. Verify that your vector is perpendicular to each of them.

**Hint** The vector product of the two vectors is perpendicular to both. To verify, show each scalar product is $0$ in full, then say what that means.

**YOUR TURN · WE DO**

**B2** Find a vector perpendicular to both $\begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$. Verify that your vector is perpendicular to each of them. [4 marks]

Answer .....


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**EXAMPLE 3 · I DO**

Find $\mathbf{a} \times \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 2 \\ -6 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 9 \\ -6 \end{pmatrix}$. What does your answer tell you about $\mathbf{a}$ and $\mathbf{b}$?

**Hint** Work out all three components. Two non-zero vectors are parallel exactly when their vector product is the zero vector.

**YOUR TURN · WE DO**

**B3** [4 marks]

$\mathbf{a} = \begin{pmatrix} 1 \\ p \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -3 \\ 6 \\ q \end{pmatrix}$ are parallel.

Use $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ to find $p$ and $q$.

Answer .....



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**C3** [4 marks]

Find the vector that is perpendicular to both $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and has $z$ component $10$.

Answer .....

**C4** [4 marks]

$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ m \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} n \\ 3 \\ 6 \end{pmatrix}$ are parallel. Use $\mathbf{a} \times \mathbf{b} = \mathbf{0}$ to find $m$ and $n$.

Answer .....


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## PART 3 The line perpendicular to two lines

**EXAMPLE 4 · I DO**

The line $L_1$ has direction $\begin{pmatrix}1\\1\\0\end{pmatrix}$ and the line $L_2$ has direction $\begin{pmatrix}2\\-1\\3\end{pmatrix}$.

Find a vector equation and a Cartesian equation of the line through $P(1, -2, 3)$ that is perpendicular to both $L_1$ and $L_2$.

**Hint** The direction you need is perpendicular to both directions: use the vector product. A simpler multiple is fine.

**YOUR TURN · WE DO**

**B4** Find a Cartesian equation of the line through $A(2, 0, -1)$ that is perpendicular to both $\begin{pmatrix}1\\2\\2\end{pmatrix}$ and $\begin{pmatrix}3\\0\\1\end{pmatrix}$. [4 marks]

Answer .....


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**EXAMPLE 5 · I DO**

The lines $L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ meet at $P$. Find a Cartesian equation of the line through $P$ perpendicular to both $L_1$ and $L_2$.

**Hint** First find $P$: two equations, then check the third. The line goes through $P$, not through either given point.

**YOUR TURN · WE DO**

**B5** The lines $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ meet at $P$. Find $P$, and a Cartesian equation of the line through $P$ perpendicular to both lines. [7 marks]

Answer .....


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**C5** Find a Cartesian equation of the line through $(-2, 1, 0)$ that is perpendicular to both of the lines $\frac{x-1}{2} = \frac{y}{-3} = \frac{z+3}{1}$ and $\frac{x+1}{1} = \frac{y-2}{1} = \frac{z}{-1}$. [5 marks]

Answer .....


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**C6** In this question you must show detailed reasoning. The lines $L_1$ : $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$ meet at the point $P$. Find a Cartesian equation of the line through $P$ that is perpendicular to both $L_1$ and $L_2$. [8 marks]

Answer .....


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**D1**

> Jo writes: "The $\mathbf{k}$ component of $\mathbf{a} \times \mathbf{b}$ is $4 - p^2$, which is $0$ when $p = 2$ or $p = -2$. So $\mathbf{a}$ and $\mathbf{b}$ are parallel for both values."

The vectors $\mathbf{a} = \begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} p \\ 4 \\ 4 \end{pmatrix}$ are given, where $p$ is a constant.

**(a)** Show that $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 4p - 8 \\ 2p - 4 \\ 4 - p^2 \end{pmatrix}$. [3 marks]

**(b)** Hence explain why Jo is wrong, and find the value of $p$ for which $\mathbf{a}$ and $\mathbf{b}$ are parallel. [3 marks]

Answer .....



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**(c)** Find the value of $p$ for which $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. [2 marks]

Answer .....

**Total for D1: 8 marks**

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**D2** A flat rectangular roof panel has a corner at $A(2, 1, 3)$, and its edges from $A$ are in the directions $\mathbf{u} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$. Units are metres, and the ground is $z = 0$. A straight pole runs from $A$ down to the ground, perpendicular to both edges.

**(a)** Find a vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$. [2 marks]

Answer .....

**(b)** Hence find a Cartesian equation of the line of the pole. [2 marks]

Answer .....



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**(c)** Find the coordinates of the point where the pole meets the ground. [2 marks]

Answer .....

**(d)** Find the acute angle between the pole and the vertical. [3 marks]

Answer .....

**Total for D2: 9 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** [8 marks]

$\mathbf{u} = \begin{pmatrix} 2 \\ 1 \\ a \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ b \\ -1 \end{pmatrix}$, where $a$ and $b$ are constants. The vector product of $\mathbf{u}$ and $\mathbf{v}$ is a non-zero multiple of $\begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$.

Find $a$ and $b$.

Answer .....



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** A vector product is perpendicular to both of the vectors it comes from. What does that tell you about $(1, -3, -1)$?

**Hint 2** Either set $\mathbf{u} \times \mathbf{v} = s(1, -3, -1)$, or use two scalar products with $(1, -3, -1)$.

**Hint 3** A multiple of $0$ gives the zero vector. Check that your $\mathbf{u} \times \mathbf{v}$ is not $\mathbf{0}$.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can work out a × b by hand, with the correct sign on the j component. | ○ ○ ○ |
| I can verify that a × b is perpendicular to a and to b with two scalar products. | ○ ○ ○ |
| I can use a × b = 0 to find when two vectors are parallel. | ○ ○ ○ |
| I can find the line through a point perpendicular to two lines, in vector and Cartesian form. | ○ ○ ○ |


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