



Index laws, zero and negative powers, and standard form

Grades 4–9 About 80 minutes No calculator

91 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Work out 2^5 .

[1 mark]

Answer

A2 Work out $(-3)^3$.

[1 mark]

Answer

A3 Work out $0.06 \times 10\,000$.

[1 mark]

Answer

A4 Work out $5^2 + 2^3$.

[1 mark]

Answer

A5 Work out $7.2 \div 0.08$.

[2 marks]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Write $3^4 \times 3^5$ as a single power of 3.

Hint 3^4 is four 3s multiplied together and 3^5 is five more. Count them all: add the indices.

B1 Write $7^6 \times 7^3 \div 7^4$ as a single power of 7. [2 marks]

Answer

EXAMPLE 2

Work out the value of 6^0 and the value of 2^{-3} .

Hint Any non-zero number to the power 0 is 1. A negative index means one over the positive power:

$$2^{-3} = \frac{1}{2^3}.$$

B2 Work out the value of $5^{-2} \times 5^4 \div 5^0$. [2 marks]

Answer

EXAMPLE 3

Simplify $(3x^4y)^2$.

Hint Square the number, and double every index:
 $(3x^4y)^2 = 3^2 \times x^{4 \times 2} \times y^2$.

B3 Simplify $\frac{12a^7b^3}{4a^2b}$. [3 marks]

Answer

EXAMPLE 4

Write 0.000 52 in standard form. Then write 3.07×10^5 as an ordinary number.

Hint Standard form is $A \times 10^n$ with $1 \leq A < 10$. Count how many places the digits move.

B4 Work out $(4 \times 10^5) \times (6 \times 10^{-2})$. [3 marks]
Give your answer in standard form.

Answer

c Practice

Grades 5–7

Each question asks for something different. Show your working.

C1 Use standard form in each part.

(a) Write 72 400 000 in standard form.

[1 mark]

Answer

(b) Write 6.05×10^{-3} as an ordinary number.

[1 mark]

Answer

(c) Write 38×10^4 in standard form.

[1 mark]

Answer

Total for C1: 3 marks

C2 Write $\frac{5^8 \times 5^{-3}}{5^2}$ as a single power of 5.

[2 marks]

Answer

C3 $3^n \times 3^4 = 3^{-2}$. Work out the value of n .

[2 marks]

Answer

C4 Simplify $(2a^3b^{-2})^4$.

[2 marks]

Answer

C5 Write these numbers in order of size. Start with the smallest.

[2 marks]

4.3×10^{-2} 0.0405 4.12×10^{-2} 0.00439

Answer

C6 Work these out. Give each answer in standard form.

(a) $(8.4 \times 10^6) \div (2.1 \times 10^{-3})$

[2 marks]

Answer

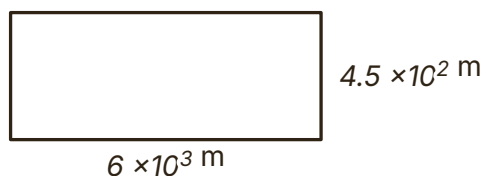
(b) $(3.2 \times 10^4) + (5.9 \times 10^3)$

[2 marks]

Answer

Total for C6: 4 marks

- C7** A field is a rectangle. Work out the area of the field. Give your answer in standard form. [2 marks]



Not drawn accurately

Answer

- C8** Write $4^3 \times 8^2$ as a single power of 2. [2 marks]

Answer

- C9** A sheet of paper is 8×10^{-3} cm thick.

- (a) Work out the height of a pile of 500 sheets, in cm. [2 marks]

Answer

- (b) A pile of these sheets is 1.2 m high. Work out the number of sheets in the pile. Give your answer in standard form. [2 marks]

Answer

Total for C9: 4 marks

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Mo says, " $(4 \times 10^3) \times (5 \times 10^4) = 20 \times 10^7$, and this answer is in standard form." [2 marks]

Explain why Mo is wrong, and write the answer in standard form.

Answer

- D2** A grain of sand has a mass of 6×10^{-4} grams.

- (a) Work out the mass of 2×10^5 grains of sand, in grams. Give your answer in standard form. [2 marks]

Answer

- (b) Work out the number of grains of sand in 3 kg of sand. Give your answer in standard form. [3 marks]

Answer

Total for D2: 5 marks

- D3** Solve $\frac{9^x}{3^{x-1}} = 27^2$. [4 marks]

Answer

D4 Show that $\frac{9^4 \times 27}{3^7} = 3^4$.

[3 marks]

D5 A country has a population of 4.5×10^7 people. On average, each person uses 1.2×10^2 litres of water a day.

- (a) Work out the total amount of water used in the country in one day, in litres. Give your answer in standard form. [2 marks]

Answer

- (b) 1 m^3 is 1000 litres. Write your answer to part (a) in m^3 . [1 mark]

Answer

- (c) A reservoir holds $2.7 \times 10^8 \text{ m}^3$ of water. How many days of water use is this? [2 marks]

Answer

Total for D5: 5 marks

D6 $8^x \times 4^y = 2^{13}$ and $\frac{9^x}{3^y} = 81$.

[6 marks]

Work out the value of x and the value of y . You must show your working.

Answer $x = \dots\dots$ $y = \dots\dots$

D7 Planet A is 1.2×10^8 km from the Sun. Planet B is 7.8×10^8 km from the Sun.

(a) How many times further from the Sun is planet B than planet A ?

[2 marks]

Answer

(b) Light travels 3×10^5 km in one second. Work out the time light takes to travel from the Sun to planet A , in seconds.

[2 marks]

Answer

Total for D7: 4 marks

D8 Simplify fully $\left(\frac{a^{-2}b^3}{a^4b^{-1}}\right)^2$.

[4 marks]

Give your answer without negative indices.

Answer

D9 Work out $\frac{3 \times 10^5}{5 \times 10^{-2}} + 4 \times 10^6$. Give your answer in standard form.

[3 marks]

Answer

D10 n is a positive whole number.

(a) Show that $2^n \times 5^{n+2} = 25 \times 10^n$.

[2 marks]

(b) Hence write $2^{12} \times 5^{14}$ in standard form.

[1 mark]

Answer

Total for D10: 3 marks

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 Solve $2^x \times 4^{x+1} = 128^2$.

[4 marks]

Answer

E2 Show that 3^{200} is greater than 2^{300} .

[3 marks]

E3 The number $5^{20} \times 2^{23}$ is written out in full as an ordinary number. How many digits does it have?

[3 marks]

Answer

E4 k is a positive whole number. Write $(4 \times 10^k)^3$ in standard form. Give your answer in terms of k . [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can multiply, divide and raise powers of the same number, using the index laws.	☐	☐	☐
I can work out zero and negative powers, and simplify expressions with letters and indices.	☐	☐	☐
I can write numbers in standard form and calculate with them without a calculator.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- | | | |
|-----------|---------------|--|
| E1 | Hint 1 | Write every number as a power of 2. |
| | Hint 2 | $4^{x+1} = 2^{2x+2}$ and $128^2 = (2^7)^2 = 2^{14}$. |
| | Hint 3 | So $x + 2x + 2 = 14$, which gives $3x = 12$ and $x = 4$. |
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- | | | |
|-----------|---------------|--|
| E2 | Hint 1 | Both indices are multiples of 100. Write each number as something to the power 100. |
| | Hint 2 | $3^{200} = (3^2)^{100}$ and $2^{300} = (2^3)^{100}$. |
| | Hint 3 | So $3^{200} = 9^{100}$ and $2^{300} = 8^{100}$. Since $9 > 8$, $9^{100} > 8^{100}$. |
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- | | | |
|-----------|---------------|---|
| E3 | Hint 1 | Pair up each 5 with a 2: $5 \times 2 = 10$. |
| | Hint 2 | $5^{20} \times 2^{23} = 5^{20} \times 2^{20} \times 2^3 = 10^{20} \times 8$. |
| | Hint 3 | 8×10^{20} is 8 followed by twenty zeros, so it has 21 digits. |
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- | | | |
|-----------|---------------|--|
| E4 | Hint 1 | Cube each part of the bracket separately. |
| | Hint 2 | $4^3 = 64$ and $(10^k)^3 = 10^{3k}$, so the value is 64×10^{3k} . |
| | Hint 3 | 64 is not between 1 and 10: $64 = 6.4 \times 10$, so the answer is $6.4 \times 10^{3k+1}$. |
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