



Circle theorems, tangents, chords and the alternate segment

Grades 4–9 About 85 minutes No calculator 90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Two angles of a triangle are 43° and 76° . Work out the third angle.

[1 mark]

Answer _____

A2 An isosceles triangle has an angle of 40° between its two equal sides. Work out the size of each of the other two angles.

[1 mark]

Answer _____

A3 Two angles on a straight line are 118° and x° . Work out x .

[1 mark]

Answer _____

A4 A right-angled triangle has shorter sides of 9 cm and 12 cm. Work out the length of the longest side.

[2 marks]

Answer _____

A5 Three angles of a quadrilateral are 95° , 70° and 110° . Work out the fourth angle.

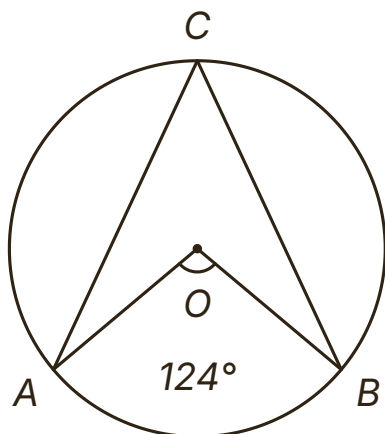
[1 mark]

Answer _____

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

A , B and C are points on a circle with centre O . C is on the major arc AB . Angle $AOB = 124^\circ$. Work out angle ACB . Give a reason.



Not drawn accurately

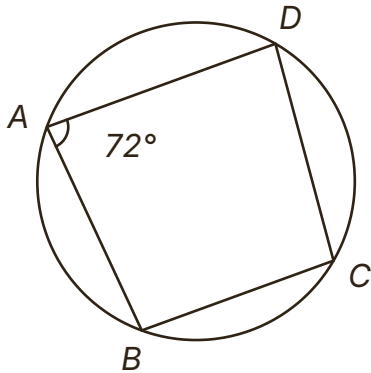
Hint The angle at the centre is twice the angle at the circumference when both stand on the same arc.

B1 A , B and C are points on a circle with centre O . C is on the major arc AB . Angle $ACB = 47^\circ$. Work out angle AOB . Give a reason. [2 marks]

Answer

EXAMPLE 2

$ABCD$ is a cyclic quadrilateral. Angle $DAB = 72^\circ$. Work out angle BCD . Give a reason.



Not drawn accurately

Hint A and C are opposite corners, and opposite angles of a cyclic quadrilateral add up to 180° .

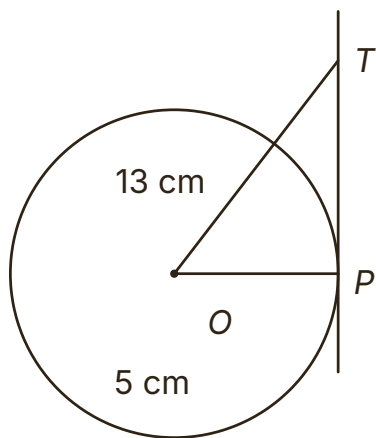
B2 $PQRS$ is a cyclic quadrilateral. [3 marks]

Angle $SPQ = 3x$ and angle $QRS = 2x + 30^\circ$. Work out the value of x . Give a reason.

Answer $x = \dots\dots$

EXAMPLE 3

TP is a tangent to a circle with centre O . P is on the circle. $OP = 5$ cm and $OT = 13$ cm. Work out the length of PT .



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Hint A tangent meets the radius at 90° , so triangle OPT is right-angled at P .

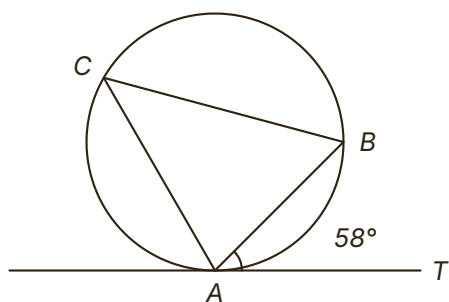
B3 TP is a tangent to a circle with centre O . P is on the circle. [2 marks]

$OP = 7$ cm and $PT = 24$ cm.
Work out the length of OT .

Answer

EXAMPLE 4

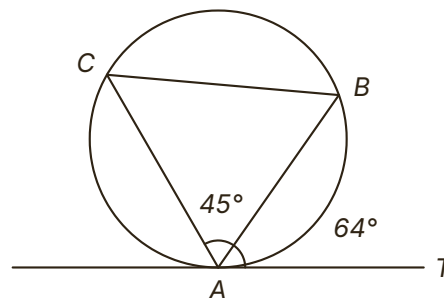
TA is a tangent to a circle at A . AB and AC are chords, and C is on the other side of AB from T . Angle $TAB = 58^\circ$. Work out angle ACB . Give a reason.



Not drawn accurately

Hint The angle between a tangent and a chord equals the angle in the alternate segment.

- B4** TA is a tangent to the circle at A . Angle $TAB = 64^\circ$ and angle $BAC = 45^\circ$. Work out angle ABC . Give reasons. [3 marks]



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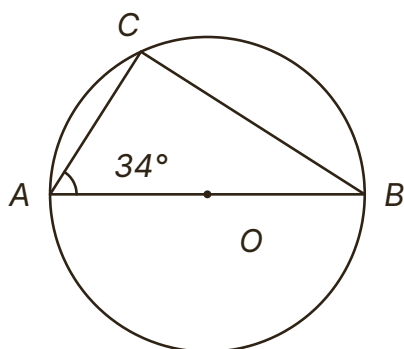
Answer

C Practice

Grades 5–7

Each question asks for something different. Show your working.

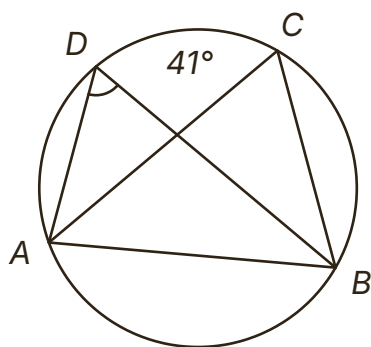
- C1** AB is a diameter of the circle with centre O . C is a point on the circle and angle $CAB = 34^\circ$. Work out angle ABC . Give reasons. [3 marks]



Not drawn accurately

Answer

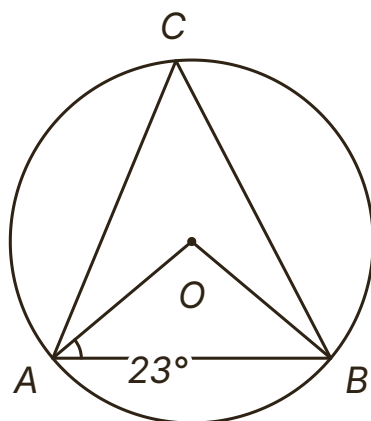
- C2** A, B, C and D are points on the circle. Angle $ADB = 41^\circ$. Work out angle ACB . Give a reason. [2 marks]



Not drawn accurately

Answer

- C3** A, B and C are points on the circle with centre O . Angle $OAB = 23^\circ$.



Not drawn accurately

- (a) Work out angle AOB . [2 marks]

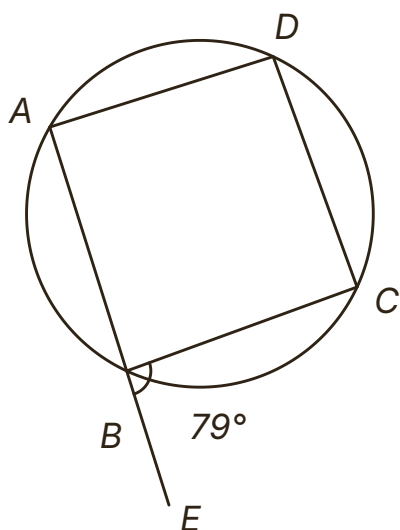
Answer

- (b) Work out angle ACB . Give a reason. [2 marks]

Answer

Total for C3: 4 marks

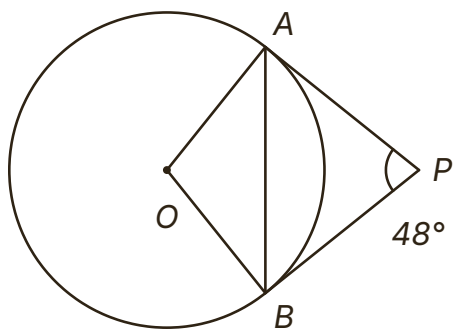
- C4** $ABCD$ is a cyclic quadrilateral. AB is extended to E . Angle $CBE = 79^\circ$. Work out angle ADC . Give reasons. [3 marks]



Not drawn accurately

Answer

- C5** PA and PB are tangents to the circle with centre O . Angle $APB = 48^\circ$.



Not drawn accurately

- (a)** Work out angle AOB . Give a reason.

[2 marks]

Answer

(b) Work out angle PAB . Give a reason.

[2 marks]

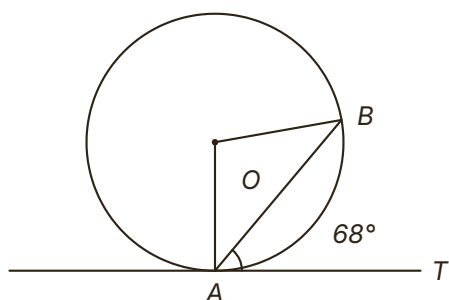
Answer

Total for C5: 4 marks

- C6** A chord of a circle is 16 cm long. The radius of the circle is 10 cm. Work out the distance from the centre of the circle to the chord. [2 marks]

Answer

- C7** TA is a tangent to the circle with centre O . AB is a chord and angle $TAB = 68^\circ$. Work out angle AOB . Give reasons. [3 marks]



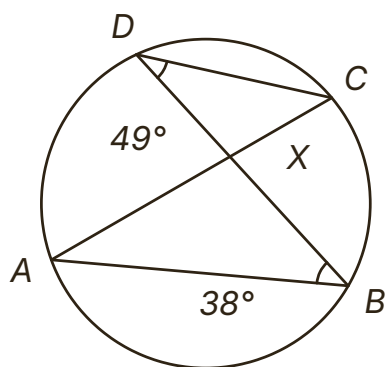
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Answer

- C8** P is a point 20 cm from the centre of a circle of radius 12 cm. A tangent is drawn from P to the circle. Work out the length of the tangent. [2 marks]

Answer

- C9** A, B, C and D are points on the circle. The chords AC and BD meet at X . Angle $ABD = 38^\circ$ and angle $BDC = 49^\circ$. Work out angle CXD . Give a reason. [3 marks]



Not drawn accurately

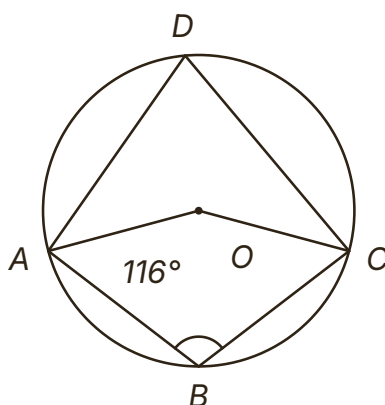
Answer

D Exam-style questions

Grades 6–9

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** A, B, C and D are points on the circle with centre O . Angle $ABC = 116^\circ$. Work out angle AOC . You must give a reason for each stage of your working. [3 marks]

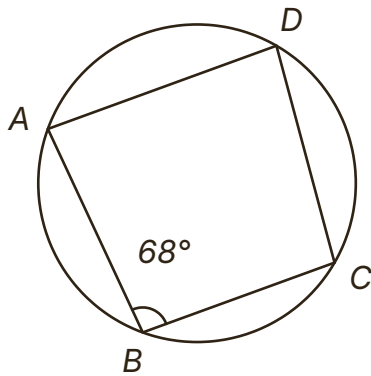


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Answer

- D2** Lily says, "Angle $ADC = 68^\circ$, because angles in the same segment are equal." [2 marks]

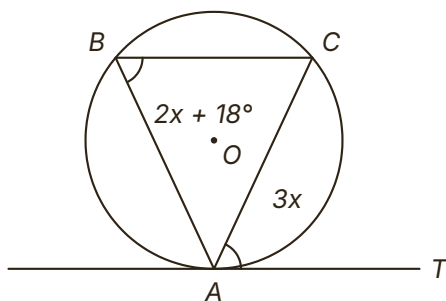
$ABCD$ is a cyclic quadrilateral and angle $ABC = 68^\circ$. Lily is wrong. Explain why, and work out angle ADC .



Not drawn accurately

Answer

- D3** TA is a tangent to the circle with centre O . A , B and C are points on the circle. Angle $TAC = 3x$ and angle $ABC = 2x + 18^\circ$.



Not drawn accurately

- (a) Work out the value of x . Give a reason. [3 marks]

Answer $x = \dots\dots$

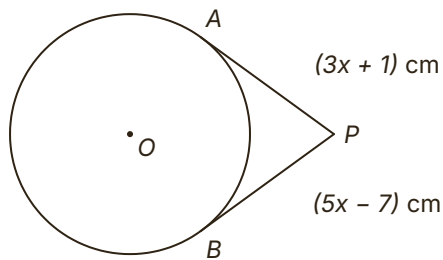
(b) Work out angle OCA . You must give a reason for each stage of your working.

[3 marks]

Answer

Total for D3: 6 marks

- D4** PA and PB are tangents to the circle. $PA = (3x + 1)$ cm and $PB = (5x - 7)$ cm. Work out the length of PA . [3 marks]



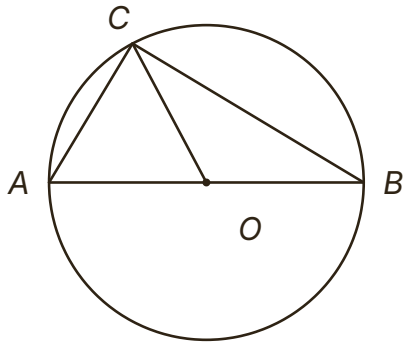
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Answer

- D5** A circle has radius 17 cm. A chord AB is 30 cm long. A second chord CD is parallel to AB , on the other side of the centre, and 23 cm from AB . Work out the length of CD . [4 marks]

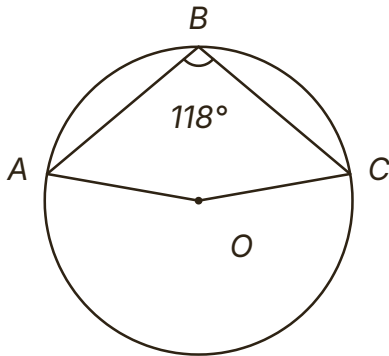
Answer

- D6** AB is a diameter of the circle with centre O , and C is a point on the circle. Prove that angle $ACB = 90^\circ$. [4 marks]



Not drawn accurately

- D7** A , B and C are points on the circle with centre O . B is on the minor arc AC and angle $ABC = 118^\circ$. Work out the reflex angle AOC . Give a reason. [3 marks]



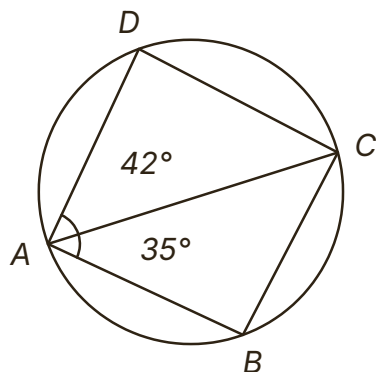
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Answer

- D8** PA is a tangent to the circle with centre O , touching it at A . Angle $OPA = 26^\circ$. Work out angle AOP . Give a reason. [2 marks]

Answer

- D9** $ABCD$ is a cyclic quadrilateral. Angle $BAC = 35^\circ$ and angle $CAD = 42^\circ$. Work out angle BCD . Give a reason. [3 marks]



Not drawn accurately

Answer

- D10** $ABCD$ is a cyclic quadrilateral. Angle $DAB = 2x + 10^\circ$, angle $BCD = 3x - 5^\circ$ and angle $ABC = 4x - 20^\circ$. Work out the size of angle ADC . [4 marks]

Answer

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

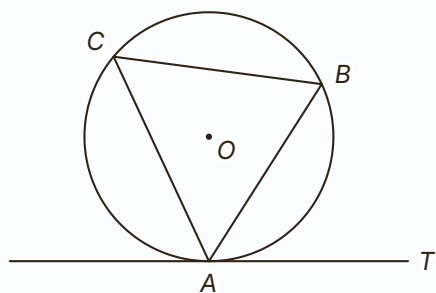
- E1** The tangents to a circle at A and at B meet at T . C is a point on the major arc AB and angle $ACB = 57^\circ$. Work out angle ATB . [3 marks]

Answer

- E2** A chord of a circle is 10 cm long. The shortest distance from the midpoint of the chord to the circle is 1 cm. Work out the radius of the circle. [4 marks]

Answer

- E3** TA is a tangent to the circle with centre O . AB is a chord and C is a point on the major arc AB . Prove that angle $TAB = \text{angle } ACB$. [4 marks]



Not drawn accurately

E4 O is the centre of a circle and AB is a chord. M is the point on AB where the line from O meets AB at 90° . Prove that $AM = MB$. [3 marks]

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use the angle at the centre, the angle in a semicircle, angles in the same segment and cyclic quadrilaterals to find angles, giving reasons.	☐	☐	☒
I can use the tangent and radius right angle, equal tangents, the perpendicular from the centre to a chord, and the alternate segment theorem.	☐	☐	☒

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1**
- Hint 1** Join A and B to the centre O . Two of the angles in quadrilateral $OATB$ are already known.
- Hint 2** Angle $AOB = 2 \times 57 = 114^\circ$, and angles OAT and OBT are both 90° .
- Hint 3** The angles of $OATB$ add up to 360° , so angle $ATB = 360 - 90 - 90 - 114 = 66^\circ$.
-
- E2**
- Hint 1** Call the radius r . The distance from the centre to the chord is $r - 1$.
- Hint 2** The perpendicular from the centre halves the chord, so $r^2 = 5^2 + (r - 1)^2$.
- Hint 3** $r^2 = 25 + r^2 - 2r + 1$, so $2r = 26$ and $r = 13$.
-
- E3**
- Hint 1** Join O to A and to B . Call angle TAB by a letter, say x .
- Hint 2** Angle $OAT = 90^\circ$, so angle $OAB = 90^\circ - x$. Triangle OAB is isosceles, so angle OBA is also $90^\circ - x$.
- Hint 3** Then angle $AOB = 180 - 2(90 - x) = 2x$, and the angle at the circumference is half of that, so angle $ACB = x$.
-
- E4**
- Hint 1** Join O to A and to B , and look at triangles OMA and OMB .
- Hint 2** $OA = OB$ (radii), OM is shared, and both triangles have a right angle at M .
- Hint 3** So the triangles are congruent (RHS), and $AM = MB$.
-