



# Quadratic graphs, factorising, the formula and simultaneous equations

Grades 4–9 About 90 minutes Calculator allowed  
98 marks

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| Name  | Class | Date  |
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|---------------------------|------------|
| <b>A</b> Before you start | Grades 4–5 |
|---------------------------|------------|

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You will need these in every question below.

**A1** Factorise  $x^2 + 9x + 14$ . [1 mark]

Answer .....

**A2** Expand and simplify  $(x - 4)(x + 9)$ . [1 mark]

Answer .....

**A3** Work out the value of  $2x^2 - 3x$  when  $x = -4$ . [2 marks]

Answer .....

**A4** Solve  $3x - 7 > 11$ . [1 mark]

Answer .....

**A5** Write down the coordinates of the point where the line  $y = 2x - 6$  crosses the  $x$ -axis. [1 mark]

Answer (....., .....)

Your teacher works through each example with you. Then try the one beside it on your own.

**EXAMPLE 1**

Solve  $x^2 - 3x - 40 = 0$ .

**Hint** Find two numbers that multiply to  $-40$  and add to  $-3$ . Then set each bracket equal to 0.

**B1** Solve  $x^2 + 3x - 28 = 0$ .

[2 marks]

Answer .....

**EXAMPLE 2**

Solve  $3x^2 + 10x - 8 = 0$  by factorising.

**Hint** The brackets start  $(3x \quad)(x \quad)$ . Try pairs of numbers that multiply to  $-8$  until the middle term is  $10x$ .

**B2** Solve  $2x^2 - 7x - 15 = 0$  by factorising.

[3 marks]

Answer .....

**EXAMPLE 3**

Solve  $2x^2 + 5x - 4 = 0$ . Give your answers correct to 2 decimal places.

**Hint** Use the quadratic formula with  $a = 2$ ,  $b = 5$  and  $c = -4$ . Work out  $b^2 - 4ac$  first.

**B3** Solve  $3x^2 - 4x - 2 = 0$ . Give your answers correct to 2 decimal places.

[3 marks]

Answer .....

**EXAMPLE 4**

Solve the simultaneous equations

$$y = x^2 - 7 \quad y = 6x$$

**Hint** Both equations say what  $y$  is, so set  $x^2 - 7 = 6x$ .  
Solve, then find each  $y$ .

**B4** Solve the simultaneous equations

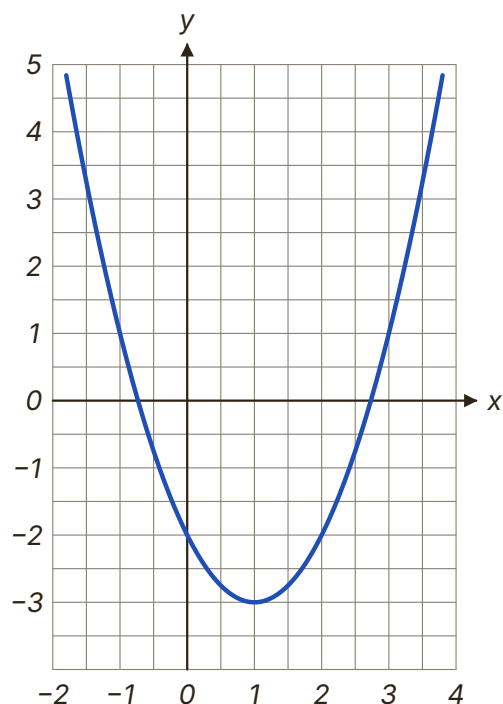
[4 marks]

$$y = x^2 + 4 \quad y = 5x + 18$$

Answer .....

Each question asks for something different. Show your working.

**C1** The graph of  $y = x^2 - 2x - 2$  is drawn on the grid.



(a) Write down the coordinates of the turning point of the curve.

[1 mark]

Answer (....., .....)

(b) Use the graph to find estimates of the solutions of  $x^2 - 2x - 2 = 0$ .

[2 marks]

Answer .....

(c) Use the graph to solve  $x^2 - 2x - 2 = 1$ .

[2 marks]

Answer .....

**Total for C1: 5 marks**

**C2** Solve  $5x^2 = 20x$ .

[2 marks]

Answer .....

- C3** Work out the value of the discriminant,  $b^2 - 4ac$ , of  $2x^2 - 3x + 5 = 0$ . [2 marks]  
Hence state how many real solutions the equation has.

Answer ..... number of solutions .....

- C4** Write  $x^2 + 6x - 11$  in the form  $(x + p)^2 + q$ . [3 marks]  
Hence write down the coordinates of the turning point of  $y = x^2 + 6x - 11$ .

Answer ..... (....., .....)

- C5** Solve  $x^2 - x - 20 \leq 0$ . [3 marks]

Answer .....

- C6** Use the quadratic formula to solve  $x^2 + 6x + 4 = 0$ . Give your answers in the form  $a \pm \sqrt{b}$ . [2 marks]

Answer .....

- C7** Solve the simultaneous equations [5 marks]

$$x^2 + y^2 = 40 \quad y = x + 4$$

Answer .....

- C8** The curve  $y = x^2 + bx + c$  crosses the  $x$ -axis at  $(1, 0)$  and  $(7, 0)$ . [3 marks]  
Find the value of  $b$ , the value of  $c$ , and the coordinates of the turning point.

Answer  $b = \dots\dots$   $c = \dots\dots$   $(\dots\dots, \dots\dots)$

- C9**  $n$  is an integer and  $n^2 < 2n + 24$ . [3 marks]  
Write down all the possible values of  $n$ .

Answer .....

## **D** Exam-style questions

Grades 6–9

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** A rectangle is  $(2x + 3)$  cm long and  $(x - 1)$  cm wide. Its area is  $33 \text{ cm}^2$ . [2 marks]  
(a) Show that  $2x^2 + x - 36 = 0$ .

- (b) Work out the perimeter of the rectangle. [3 marks]

Answer .....

Total for D1: 5 marks

**D2** Lena says, "The solution of  $x^2 > 16$  is  $x > 4$ ." [2 marks]

Lena is not right. Explain why, and write down the full solution.

Answer .....

**D3** A ball is thrown from the top of a wall. Its height,  $h$  metres, above the ground  $t$  seconds later is [3 marks]

$$h = 2 + 14t - 5t^2$$

Work out the time it takes for the ball to reach the ground. Give your answer correct to 3 significant figures.

Answer .....

**D4** Show that the line  $y = 2x - 1$  touches the curve  $y = x^2$  at exactly one point, and find the coordinates of that point. [3 marks]

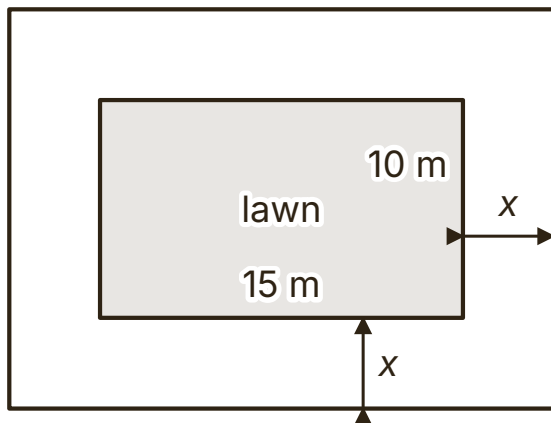
Answer (....., .....)

**D5** Solve  $\frac{8}{x+1} - \frac{3}{x} = 1$ . [4 marks]

You must show your working.

Answer .....

- D6** A rectangular lawn is 15 m long and 10 m wide. A path of the same width,  $x$  metres, goes all the way round the lawn. The area of the path is  $84 \text{ m}^2$ .



Not drawn accurately

- (a)** Show that  $2x^2 + 25x - 42 = 0$ . [3 marks]

- (b)** Work out the width of the path. You must show your working. [3 marks]

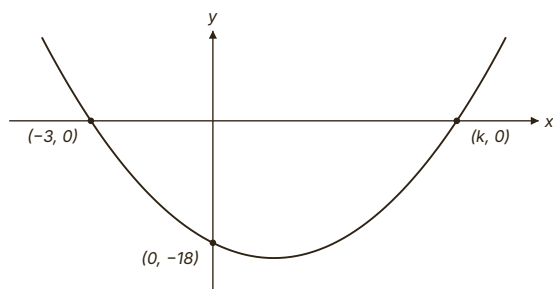
Answer .....

Total for D6: 6 marks

- D7** Solve  $2x^2 - x - 15 > 0$ . [3 marks]

Answer .....

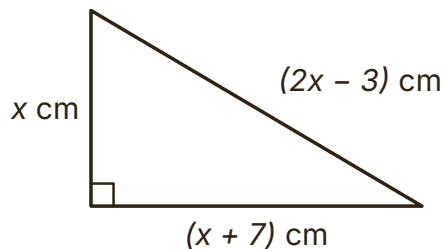
- D8** The diagram shows a sketch of the curve  $y = x^2 + bx + c$ . [3 marks]  
The curve meets the  $y$ -axis at  $(0, -18)$  and the  $x$ -axis at  $(-3, 0)$  and  $(k, 0)$ .  
Find the value of  $b$  and the value of  $k$ .



Not drawn accurately

Answer  $b = \dots\dots$   $k = \dots\dots$

- D9** The sides of a right-angled triangle are  $x$  cm,  $(x + 7)$  cm and  $(2x - 3)$  cm. The longest side is  $(2x - 3)$  cm. [4 marks]  
Work out the value of  $x$ . Give your answer correct to 2 decimal places.



Not drawn accurately

Answer  $x = \dots\dots$

**D10** Solve the simultaneous equations

[5 marks]

$$y = x^2 - 4 \quad x + y = 3$$

Give your values of  $x$  and  $y$  correct to 2 decimal places.

Answer .....

**E** **Extension**

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

**E1** Find the values of  $k$  for which the equation  $x^2 + (k + 1)x + 4 = 0$  has no real solutions.

[4 marks]

Answer .....

**E2** The sum of the whole numbers from 1 to  $n$  is  $\frac{1}{2}n(n + 1)$ .  
Find the smallest value of  $n$  for which this sum is more than 500.

[3 marks]

Answer .....

**E3** Solve  $x - 7\sqrt{x} + 12 = 0$ .

[3 marks]

Answer .....

**E4** The line  $y = x + c$  is a tangent to the circle  $x^2 + y^2 = 8$ .

[4 marks]

Find the possible values of  $c$ .

Answer .....

### **F** How did it go?

Colour one circle on each line.

|                                                                                           | Not yet               | Nearly                | Yes                   |
|-------------------------------------------------------------------------------------------|-----------------------|-----------------------|-----------------------|
| I can read the roots, the intercept and the turning point off a quadratic graph.          | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| I can solve a quadratic by factorising, including when the coefficient of $x^2$ is not 1. | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| I can use the quadratic formula and the discriminant.                                     | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| I can complete the square to find a turning point.                                        | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| I can solve a linear and a quadratic equation together.                                   | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |
| I can solve a quadratic inequality.                                                       | <input type="radio"/> | <input type="radio"/> | <input type="radio"/> |

## Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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|           |               |                                                                                     |
|-----------|---------------|-------------------------------------------------------------------------------------|
| <b>E1</b> | <b>Hint 1</b> | An equation has no real solutions when its discriminant, $b^2 - 4ac$ , is negative. |
|           | <b>Hint 2</b> | $(k + 1)^2 - 16 < 0$                                                                |
|           | <b>Hint 3</b> | $(k + 1)^2 < 16$ , so $-4 < k + 1 < 4$ , which gives $-5 < k < 3$ .                 |

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|           |               |                                                                                                                              |
|-----------|---------------|------------------------------------------------------------------------------------------------------------------------------|
| <b>E2</b> | <b>Hint 1</b> | Write an inequality and turn it into a quadratic.                                                                            |
|           | <b>Hint 2</b> | $n^2 + n - 1000 > 0$                                                                                                         |
|           | <b>Hint 3</b> | The positive root of $n^2 + n - 1000 = 0$ is about 31.1, so $n = 32$ . Check: the sum to 31 is 496 and the sum to 32 is 528. |

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|           |               |                                                                                                        |
|-----------|---------------|--------------------------------------------------------------------------------------------------------|
| <b>E3</b> | <b>Hint 1</b> | $x$ is $(\sqrt{x})^2$ . Give $\sqrt{x}$ a letter of its own.                                           |
|           | <b>Hint 2</b> | With $y = \sqrt{x}$ : $y^2 - 7y + 12 = 0$ , so $(y - 3)(y - 4) = 0$ .                                  |
|           | <b>Hint 3</b> | $\sqrt{x} = 3$ gives $x = 9$ , and $\sqrt{x} = 4$ gives $x = 16$ . Both work in the original equation. |

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|           |               |                                                                                                               |
|-----------|---------------|---------------------------------------------------------------------------------------------------------------|
| <b>E4</b> | <b>Hint 1</b> | Substitute the line into the circle. A tangent meets the circle at exactly one point.                         |
|           | <b>Hint 2</b> | $2x^2 + 2cx + c^2 - 8 = 0$                                                                                    |
|           | <b>Hint 3</b> | One point means the discriminant is 0: $4c^2 - 8(c^2 - 8) = 0$ , so $64 - 4c^2 = 0$ and $c = 4$ or $c = -4$ . |

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