



Pythagoras' theorem and trigonometry in right-angled triangles

Grades 4–9 About 85 minutes Calculator allowed

93 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Work out 6.5^2 .

[1 mark]

Answer _____

A2 $x^2 = 196$ and x is positive. Find x .

[1 mark]

Answer _____

A3 Solve $\frac{x}{7} = 0.6$.

[1 mark]

Answer _____

A4 Solve $\frac{12}{x} = 0.8$.

[2 marks]

Answer _____

A5 Round 23.4567 to 3 significant figures.

[1 mark]

Answer _____

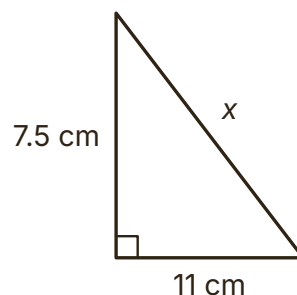
Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

In triangle ABC , angle $B = 90^\circ$, $AB = 8$ cm and $BC = 13$ cm. Work out the length of AC . Give your answer to 1 decimal place.

Hint AC is the hypotenuse: it is opposite the right angle. Square the two shorter sides and add.

B1 Work out the length x . Give your answer to 1 decimal place. [3 marks]



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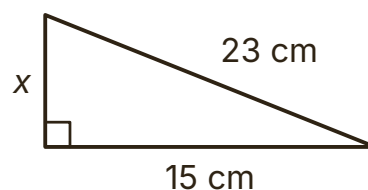
Answer

EXAMPLE 2

In triangle PQR , angle $Q = 90^\circ$, $PR = 17.2$ cm and $PQ = 9.8$ cm. Work out the length of QR . Give your answer to 1 decimal place.

Hint This time the hypotenuse is given. Square it and take away the square of the other side.

B2 Work out the length x . Give your answer to 1 decimal place. [3 marks]



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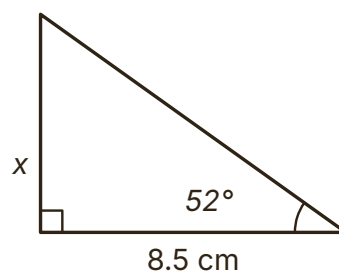
Answer

EXAMPLE 3

In triangle ABC , angle $B = 90^\circ$, angle $A = 38^\circ$ and $AC = 12$ cm. Work out the length of BC .
Give your answer to 3 significant figures.

Hint Label the sides from the angle you know: BC is opposite, AC is the hypotenuse. Opposite and hypotenuse means sine.

B3 Work out the length x . Give your answer to 3 significant figures. [3 marks]



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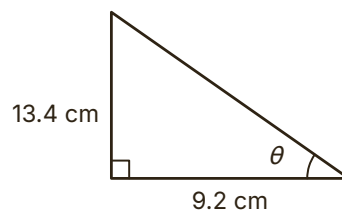
Answer

EXAMPLE 4

In triangle DEF , angle $E = 90^\circ$, $DF = 11$ cm and $EF = 7$ cm. Work out the size of angle D .
Give your answer to 1 decimal place.

Hint EF is opposite angle D and DF is the hypotenuse. Write $\sin D = \frac{7}{11}$, then use the inverse sine.

B4 Work out the size of angle θ . Give your answer to 1 decimal place. [3 marks]

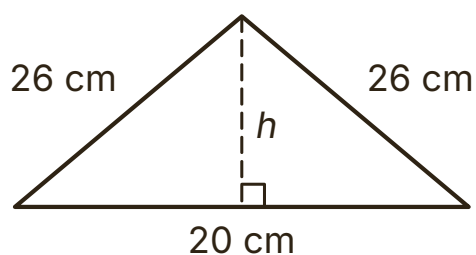


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Answer

Each question asks for something different. Show your working.

C1 The diagram shows an isosceles triangle. The dashed line is its height h .



Not drawn accurately

(a) Work out h .

[2 marks]

Answer

(b) Work out the area of the triangle.

[2 marks]

Answer

Total for C1: 4 marks

C2 A triangle has sides of length 9 cm, 12 cm and 16 cm.

[2 marks]

Show that the triangle is not right-angled.

C3 A is the point $(-2, 7)$ and B is the point $(6, -8)$.

[3 marks]

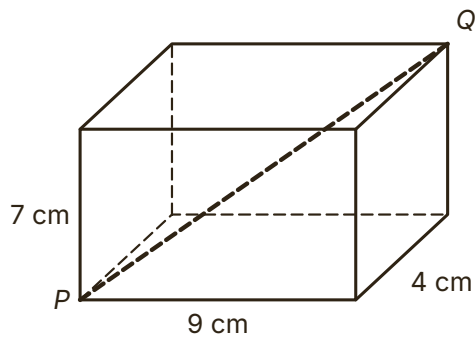
Work out the length of the line segment AB .

Answer

- C4** The diagram shows a cuboid 9 cm long, 4 cm wide and 7 cm high.

[3 marks]

Work out the length of the diagonal PQ . Give your answer to 3 significant figures.

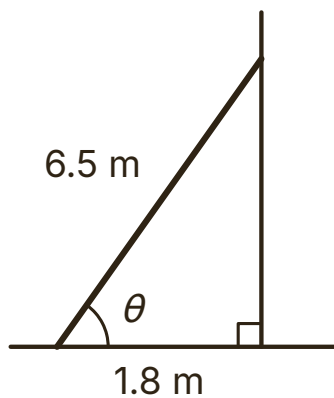


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Answer

- C5** A ladder 6.5 m long leans against a vertical wall. The foot of the ladder is 1.8 m from the wall on level ground. [3 marks]

Work out the angle θ between the ladder and the ground. Give your answer to 1 decimal place.

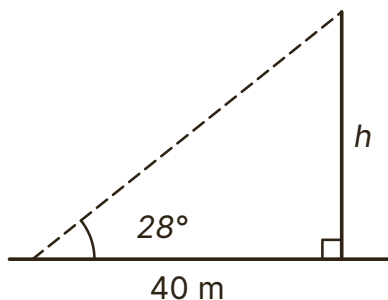


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Answer

- C6** Leah stands 40 m from the foot of a vertical tower on level ground. The angle of elevation of the top of the tower from where she stands is 28° . [3 marks]

Work out the height h of the tower. Give your answer to 3 significant figures.



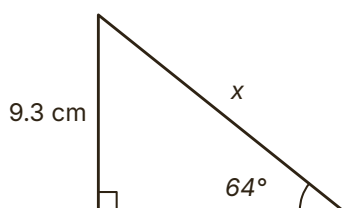
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Answer

- C7** A boat sails 12 km due north from a harbour H , and then 7 km due east to a buoy B . [3 marks]
Work out the bearing of B from H . Give your answer to the nearest degree.

Answer

- C8** Work out the length x . Give your answer to 3 significant figures. [3 marks]



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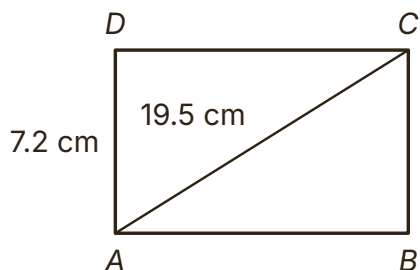
Answer

Answer every question in the space given. Show your working: most of the marks are for method.

D1 $ABCD$ is a rectangle. $AD = 7.2$ cm and the diagonal $AC = 19.5$ cm.

[3 marks]

Work out the perimeter of the rectangle. Give your answer to 3 significant figures.



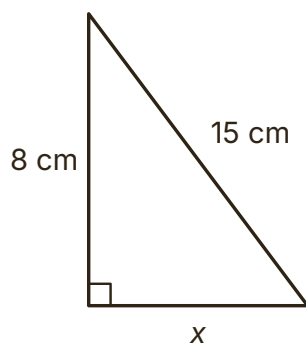
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Answer

D2 Ravi works out the length x . He writes " $x = \sqrt{8^2 + 15^2} = 17$ cm".

[3 marks]

Explain the mistake Ravi has made, and work out the correct value of x . Give your answer to 3 significant figures.



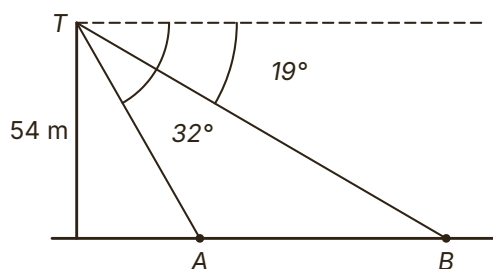
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Answer

- D3** T is the top of a vertical cliff 54 m high. Two boats, A and B , are in a straight line out to sea from the foot of the cliff. [4 marks]

The angle of depression of A from T is 32° . The angle of depression of B from T is 19° .

Work out the distance AB . Give your answer to 3 significant figures.



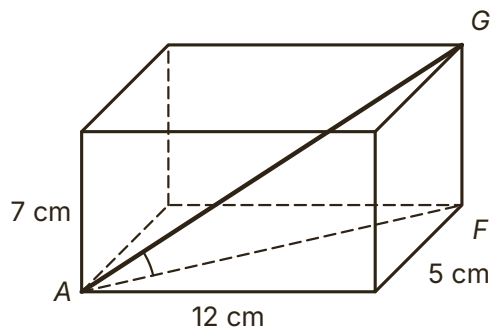
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Answer

- D4** The diagram shows a cuboid 12 cm long, 5 cm wide and 7 cm high.

[4 marks]

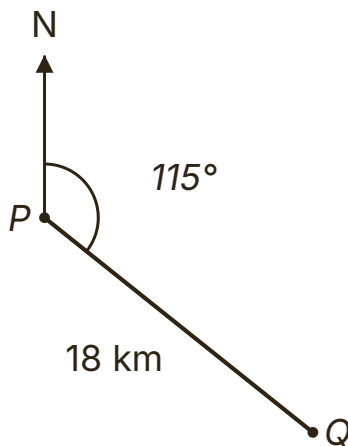
Work out the size of angle GAF , the angle between the diagonal AG and the base of the cuboid. Give your answer to 1 decimal place.



Not drawn accurately

Answer

- D5** A ship sails 18 km from a port P to a point Q on a bearing of 115° .



Not drawn accurately

- (a)** Work out how far east of P the point Q is. Give your answer to 3 significant figures. [2 marks]

Answer

- (b) Work out how far south of P the point Q is. Give your answer to 3 significant figures. [2 marks]

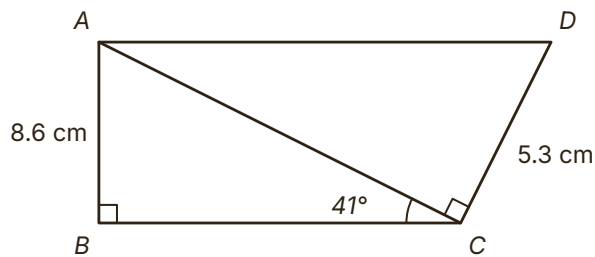
Answer

- (c) Write down the bearing of P from Q . [1 mark]

Answer

Total for D5: 5 marks

- D6** The diagram shows a quadrilateral $ABCD$. Angle $ABC = 90^\circ$ and angle $ACD = 90^\circ$.
 $AB = 8.6$ cm, $CD = 5.3$ cm and angle $ACB = 41^\circ$.



Not drawn accurately

- (a) Work out the length of AD . Give your answer to 3 significant figures. [4 marks]

Answer

- (b) Work out the size of angle CAD . Give your answer to 1 decimal place. [2 marks]

Answer

Total for D6: 6 marks

- D7** A triangle has sides of length 2.8 cm, 4.5 cm and 5.3 cm.
Show that the triangle is right-angled.

[2 marks]

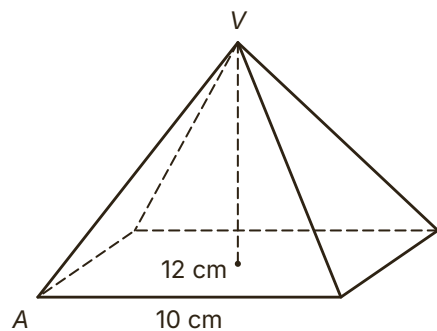
- D8** A ladder 7.2 m long leans against a vertical wall on level ground. At first the ladder makes an angle of 68° with the ground. The foot of the ladder is then moved closer to the wall, so that the angle becomes 75° .
How much higher up the wall does the top of the ladder reach? Give your answer in centimetres to the nearest centimetre.

[4 marks]

Answer

- D9** A pyramid has a square base of side 10 cm. Its top V is vertically above the centre of the base, and its height is 12 cm.
Work out the length of the sloping edge VA . Give your answer to 3 significant figures.

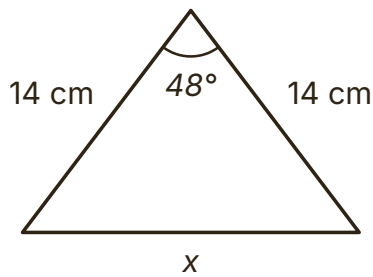
[4 marks]



Not drawn accurately

Answer

- D10** An isosceles triangle has two sides of length 14 cm. The angle between them is 48° . [3 marks]
Work out the length x of the third side. Give your answer to 3 significant figures.



Not drawn accurately

Answer

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** The diagonal from one corner of a cube to the opposite corner is 15 cm long. [3 marks]
Work out the total surface area of the cube.

Answer

E2 In triangle ABC , angle $B = 90^\circ$ and $AC = 30$ cm. The tangent of angle A is 0.75. [3 marks]

Work out the perimeter of the triangle.

Answer

E3 Two points P and Q are 30 m apart on level ground, in a straight line with the foot of a vertical mast. The angle of elevation of the top of the mast is 26° from P and 41° from Q . [4 marks]

Work out the height of the mast. Give your answer to 3 significant figures.

Answer

E4 A box is a cuboid with a base 60 cm by 45 cm. A thin rod 110 cm long must fit inside the box. [3 marks]

Work out the smallest whole number of centimetres the height of the box can be.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use Pythagoras' theorem to find the hypotenuse or a shorter side, and to test whether a triangle is right-angled.	☐	☐	☐
I can use Pythagoras' theorem twice to find a length inside a cuboid or a pyramid.	☐	☐	☐
I can choose sine, cosine or tangent and use it to find a side or an angle.	☐	☐	☐
I can solve problems with angles of elevation and depression, bearings and isosceles triangles.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1** **Hint 1** Call the edge of the cube s . Use Pythagoras twice to write the long diagonal in terms of s .
- Hint 2** $s^2 + s^2 + s^2 = 15^2$
- Hint 3** $3s^2 = 225$, so $s^2 = 75$. Each of the six faces has area s^2 , so the surface area is $6 \times 75 = 450 \text{ cm}^2$.
-
- E2** **Hint 1** $\tan A = 0.75 = \frac{3}{4}$. Draw a right-angled triangle with sides 3 and 4 next to angle A .
- Hint 2** That triangle has hypotenuse 5, so triangle ABC is 6 times bigger.
- Hint 3** $BC = 18 \text{ cm}$ and $AB = 24 \text{ cm}$, so the perimeter is $18 + 24 + 30 = 72 \text{ cm}$.
-
- E3** **Hint 1** Call the height h . Write the distance from each point to the foot of the mast in terms of h .
- Hint 2** $\frac{h}{\tan 26^\circ} - \frac{h}{\tan 41^\circ} = 30$
- Hint 3** $h \left(\frac{1}{\tan 26^\circ} - \frac{1}{\tan 41^\circ} \right) = 30$, so $h = \frac{30}{2.0503 - 1.1504} = 33.3 \text{ m}$.
-
- E4** **Hint 1** The longest rod that fits runs from one corner to the opposite corner.
- Hint 2** $60^2 + 45^2 + h^2 = 110^2$
- Hint 3** $h^2 = 6475$, so $h = 80.47 \dots$. The height must be at least this, so the smallest whole number of centimetres is 81.