



Similar triangles, and the area and volume of similar shapes

Grades 4–9 About 80 minutes Calculator allowed 90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 Write the ratio 15 : 24 in its simplest form.

[1 mark]

Answer _____

A2 Work out 2.5^3 .

[1 mark]

Answer _____

A3 Work out $\sqrt[3]{343}$.

[1 mark]

Answer _____

A4 Solve $\frac{x}{6} = \frac{9}{4}$.

[2 marks]

Answer _____

A5 Two angles of a triangle are 49° and 73° . Work out the third angle.

[1 mark]

Answer _____

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

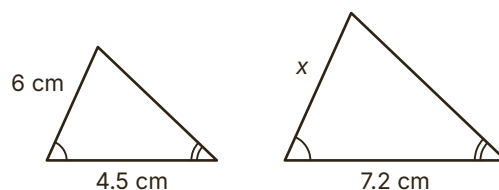
Triangles ABC and PQR are similar. AB corresponds to PQ and BC corresponds to QR .

$AB = 6$ cm, $PQ = 15$ cm and $BC = 8$ cm.

Work out the length of QR .

Hint Find the scale factor from the pair of sides you know both of: $15 \div 6$. Multiply the other side by it.

- B1** The two triangles are similar. [2 marks]
Work out the length x .



Not drawn accurately

Answer

EXAMPLE 2

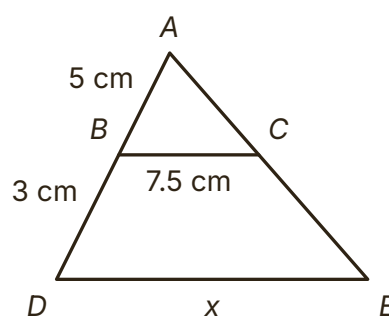
In triangle ADE , B is on AD and C is on AE . BC is parallel to DE .

$AB = 4$ cm, $AD = 10$ cm and $BC = 6$ cm.

Work out the length of DE .

Hint BC parallel to DE makes triangle ABC similar to triangle ADE . Compare AB with the WHOLE side AD , not with BD .

- B2** BC is parallel to DE . Work out the length x . [3 marks]



Not drawn accurately

Answer

EXAMPLE 3

Two shapes are similar. A side of the smaller shape is 3 cm and the matching side of the larger shape is 7.5 cm. The area of the smaller shape is 24 cm^2 . Work out the area of the larger shape.

Hint The length scale factor is 2.5. Areas are multiplied by the square of it.

- B3** Two triangles are similar. A side of the smaller triangle is 6 cm and the matching side of the larger triangle is 9 cm. The area of the larger triangle is 81 cm^2 .
Work out the area of the smaller triangle. [3 marks]

Answer

EXAMPLE 4

Two bottles are similar. Their volumes are 250 cm^3 and 2000 cm^3 . The smaller bottle is 12 cm tall. Work out the height of the larger bottle.

Hint The volume scale factor is $2000 \div 250 = 8$. The length scale factor is the cube root of it.

- B4** Two cones are similar. Their volumes are 135 cm^3 and 625 cm^3 . The larger cone is 20 cm tall.
Work out the height of the smaller cone. [3 marks]

Answer

C Practice

Grades 5–7

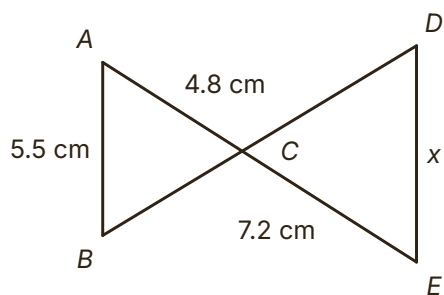
Each question asks for something different. Show your working.

- C1** Triangle P has angles of 48° and 77° . Triangle Q has angles of 77° and 55° . [2 marks]
Show that the two triangles are similar.

C2 AB is parallel to DE . The lines AE and BD cross at C .

[2 marks]

Work out the length x .



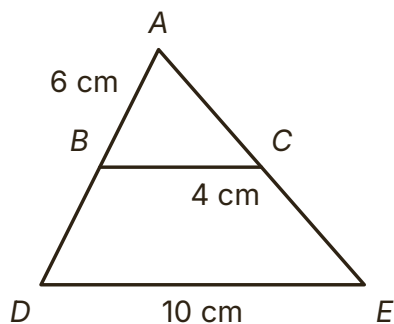
Not drawn accurately

Answer

C3 BC is parallel to DE .

[3 marks]

Work out the length of BD .



Not drawn accurately

Answer

- C4** Two triangles are similar. Their areas are 32 cm^2 and 72 cm^2 . The shortest side of the smaller triangle is 6 cm. [3 marks]

Work out the shortest side of the larger triangle.

Answer

- C5** Two solids are similar. Their heights are 5 cm and 8 cm. The volume of the smaller solid is 75 cm^3 . [2 marks]

Work out the volume of the larger solid.

Answer

- C6** Two cylinders are similar. Their volumes are 54 cm^3 and 250 cm^3 . The surface area of the smaller cylinder is 90 cm^2 . [3 marks]

Work out the surface area of the larger cylinder.

Answer

- C7** A model of a van is made to a scale of 1 : 40.

- (a) The model is 11 cm long. Work out the length of the real van in metres. [1 mark]

Answer

- (b) The windscreen of the real van has an area of 1.28 m^2 . Work out the area of the windscreen on the model in cm^2 . [2 marks]

Answer

- (c) The load space of the model has a volume of 1.5 cm^3 . Work out the volume of the load space of the real van in m^3 . [2 marks]

Answer

Total for C7: 5 marks

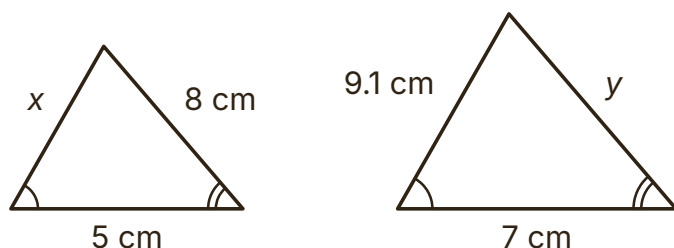
- C8** A post 1.6 m tall casts a shadow 2.4 m long. At the same time, a tree casts a shadow 15 m long on the same level ground. [2 marks]
Work out the height of the tree.

Answer

- C9** Rectangle R is 6 cm by 9 cm. Rectangle S is 10 cm by 14 cm. [2 marks]
Show that the two rectangles are not similar.

Answer every question in the space given. Show your working: most of the marks are for method.

D1 The two triangles are similar. Matching angles are marked the same.



Not drawn accurately

(a) Work out the length x .

[2 marks]

Answer

(b) Work out the length y .

[2 marks]

Answer

Total for D1: 4 marks

D2 Jo says, "The sides of shape B are twice as long as the sides of shape A . Shape A has an area of 18 cm^2 , so shape B has an area of $2 \times 18 = 36 \text{ cm}^2$."

[3 marks]

Shapes A and B are similar. Explain why Jo is wrong, and work out the area of shape B .

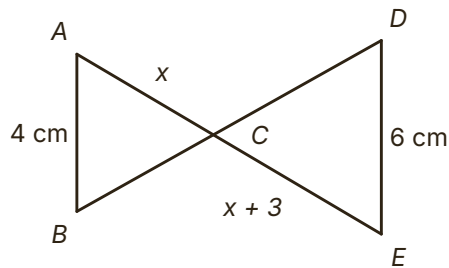
Answer

- D3** AB is parallel to DE . The lines AE and BD cross at C .

[3 marks]

$AB = 4$ cm, $DE = 6$ cm, $AC = x$ cm and $CE = (x + 3)$ cm.

Work out the value of x . You must show your working.



Not drawn accurately

Answer

- D4** Two jugs are similar. The smaller jug holds 0.4 litres and the larger jug holds 1.35 litres. [3 marks]
The larger jug is 21 cm tall.

Work out the height of the smaller jug.

Answer

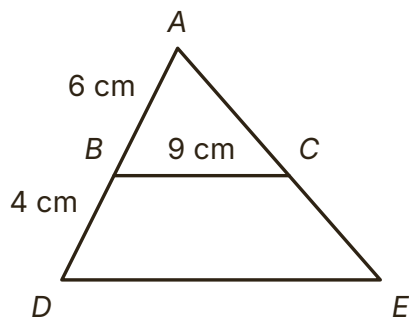
- D5** Two solids are similar. Their surface areas are 48 cm^2 and 108 cm^2 . The volume of the [4 marks]
smaller solid is 64 cm^3 .

Work out the volume of the larger solid.

Answer

D6 B is on AD and C is on AE . BC is parallel to DE .

$AB = 6$ cm, $BD = 4$ cm and $BC = 9$ cm. The area of triangle ABC is 27 cm².



Not drawn accurately

(a) Work out the length of DE .

[2 marks]

Answer

(b) Work out the area of the shape $BDEC$.

[4 marks]

Answer

Total for D6: 6 marks

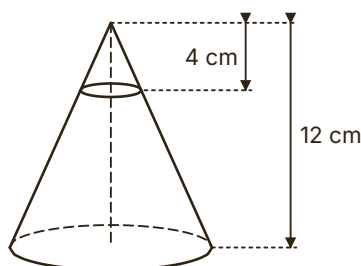
D7 Two statues are similar. One is 30 cm tall and the other is 75 cm tall. The smaller statue needs 0.2 litres of paint to cover it. [3 marks]

Paint is sold in tins of 0.5 litres. How many tins are needed to paint the larger statue?

Answer

- D8** A solid cone is 12 cm tall and has a volume of 540 cm^3 . A small cone 4 cm tall is cut off the top, with the cut parallel to the base. The part that is left is called a frustum. [3 marks]

Work out the volume of the frustum.



Not drawn accurately

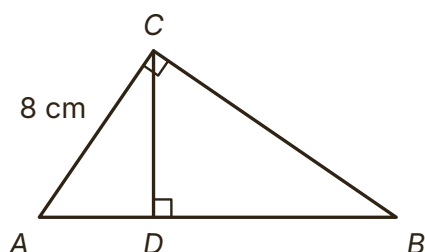
Answer

- D9** A shop sells two similar posters. One is 40 cm wide and the other is 60 cm wide. The price of a poster is in proportion to its area. The smaller poster costs £3.20. [3 marks]

The shop sells the larger poster for £7.50. Is this the correct price? You must show your working.

Answer

- D10** Triangle ABC has a right angle at C . D is the point on AB where CD is perpendicular to AB . $AC = 8 \text{ cm}$ and $AB = 10 \text{ cm}$.



Not drawn accurately

- (a) Explain why triangle ACD is similar to triangle ABC . [1 mark]

(b) Work out the length of AD .

[3 marks]

Answer

Total for D10: 4 marks

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 A rectangle is cut in half, parallel to its shorter side. Each half is similar to the whole rectangle. [3 marks]

Work out the ratio of the longer side to the shorter side of the rectangle.

Answer

E2 Two solids are similar. Their volumes are 250 cm^3 and 432 cm^3 . The surface area of the larger solid is 44 cm^2 more than the surface area of the smaller solid. [4 marks]

Work out the surface area of the smaller solid.

Answer

- E3** In triangle ABC , $BC = 12$ cm. A line DE , parallel to BC , cuts the triangle into two parts of equal area: a triangle ADE and the part below it. [3 marks]
- Work out the length of DE . Give your answer to 3 significant figures.

Answer

- E4** Two similar triangles have perimeters of 24 cm and 36 cm. The area of the smaller triangle is 20 cm^2 . [3 marks]
- Work out the total area of the two triangles.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can show that two triangles are similar, and find a missing length using the scale factor.	☐	☐	☐
I can find lengths in triangles cut by a parallel line, including where two lines cross.	☐	☐	☐
I can use the area scale factor k^2 and the volume scale factor k^3 , and work back from them to k .	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- E1** **Hint 1** Call the shorter side 1 and the longer side x . Write the two sides of one half.
- Hint 2** One half is 1 by $\frac{x}{2}$, so its longer side is 1 and its shorter side is $\frac{x}{2}$.
- Hint 3** Similar means $\frac{x}{1} = \frac{1}{x/2}$, so $x^2 = 2$ and $x = \sqrt{2}$.
-
- E2** **Hint 1** Find the length scale factor from the volumes first.
- Hint 2** $\frac{432}{250} = 1.728 = 1.2^3$, so the area scale factor is $1.2^2 = 1.44$.
- Hint 3** If the smaller surface area is S , then $1.44S - S = 44$, so $0.44S = 44$ and $S = 100$.
-
- E3** **Hint 1** Triangle ADE is similar to triangle ABC . What fraction of the area of ABC is it?
- Hint 2** The area scale factor from ADE to ABC is 2.
- Hint 3** So the length scale factor is $\sqrt{2}$, and $DE = \frac{12}{\sqrt{2}} = 8.49$ cm.
-
- E4** **Hint 1** The perimeter is a length, so the perimeters give the length scale factor.
- Hint 2** Length scale factor $36 \div 24 = 1.5$, so the area scale factor is 2.25.
- Hint 3** The larger area is $20 \times 2.25 = 45$, so the total is $20 + 45 = 65$ cm².
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