



The binomial and normal distributions

About 80 minutes

Calculator allowed 84 marks

Date _____

A Do Now

The first two are what this chapter needs. The rest bring back earlier chapters so nothing is forgotten.

A1 Needed today [2 marks]

Each day, the probability that the lift breaks down is 0.45, independently of other days. Find the probability that the lift breaks down on at least one of the next 4 days. Give your answer to 3 decimal places.

Answer

A2 Needed today [2 marks]

Ella's distance in the shot put had a standardised score of -2.1 . What does this tell you about Ella's distance compared with the rest of the group?

A3 Maths skill [2 marks]

A number x is truncated to 2 decimal places. The result is 2.40. Write down the error interval for x .

Answer

Given that $16 \times 17 = 272$, work out $2.72 \div 1.7$.

Answer

Work out $\sqrt{30^2 - 24^2}$.

Answer

B Examples, then your turn

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

Each phone screen made by a factory has a fault with probability 0.02. 20 screens are checked. X is the number of screens with a fault. Give two conditions for X to have a binomial distribution, in context.

Hint Say what the trials are, and what must stay the same from one trial to the next.

- B1** A quiz has 10 questions, each with four options. Ali guesses every answer. X is the number of answers he gets right. Give the conditions, in context, that let X be modelled as binomial, and write down the values of n and p . [3 marks]

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EXAMPLE 2

$X \sim B(6, 0.25)$. Find $P(X = 2)$, to 3 decimal places.

Hint Count the arrangements of 2 successes in 6 trials.

- B2** $X \sim B(7, 0.1)$.
- (a) Find $P(X = 1)$, correct to 3 decimal places. [2 marks]

Answer

- (b) Find $P(X \geq 2)$, correct to 3 decimal places. [2 marks]

Answer

Total for B2: 4 marks

EXAMPLE 3

In families with 5 children, the number of girls, X , is modelled by $B(5, 0.5)$. (a) Work out the mean of X . (b) In a survey of 160 such families, estimate how many have exactly 3 girls.

Hint The mean is np . Then probability $\times$ number of families.

B3 10% of the jars filled by a machine are underfilled. The jars are packed in boxes of 3. X is the number of underfilled jars in a box.

(a) Work out the mean of X . [1 mark]

Answer

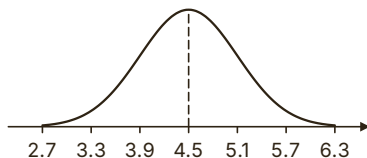
(b) There are 1000 boxes. [2 marks]
Estimate how many of them have no underfilled jars.

Answer

Total for B3: 3 marks

EXAMPLE 4

The masses of new-born lambs are normally distributed with mean 4.5 kg and standard deviation 0.6 kg. (a) Estimate the percentage of lambs with a mass between 2.7 kg and 3.9 kg. (b) A farm has 400 new-born lambs. Estimate how many have a mass in this range.



Hint 2.7 is 3 sd below the mean and 3.9 is 1 sd below.

B4 The battery lives of a make of torch are normally distributed with mean 30 hours and standard deviation 2.5 hours. A shop sells 2000 of the torches. Estimate how many of them have a battery life between 32.5 and 37.5 hours.

Answer

EXAMPLE 5

Two machines fill bags labelled 1000 g. The masses from machine *A* are normal with mean 1008 g and standard deviation 4 g. The masses from machine *B* are normal with mean 1010 g and standard deviation 10 g. Which machine is more likely to fill a bag with less than 1000 g? Give a reason.

Hint How many standard deviations below each mean is 1000 g?

B5 Two runners' times for 400 m are normally distributed. [3 marks]

Ana: mean 61 seconds, standard deviation 1 second.
Bea: mean 60 seconds, standard deviation 2 seconds. The team needs a time under 58 seconds. Which runner is more likely to run under 58 seconds? Give a reason.

C Practice

Each question asks for something different. Show your working.

- C1** A spinner lands on red with probability 0.3. It is spun 9 times. Work out the probability of exactly 4 reds. Give your answer to 3 decimal places. [2 marks]

Answer

- C2** A treatment works for 85% of patients, independently. 6 patients have the treatment. Find the probability that it works for at most 4 of them. Give your answer to 3 decimal places. [3 marks]

Answer

- C3** On any day, the probability that a train is cancelled is 0.05, independently of other days.

- (a) Over how many days would you expect 3 cancellations? [1 mark]

Answer

- (b) Work out the probability of no cancellations in 10 days. Give your answer to 3 decimal places. [1 mark]

Answer

Total for C3: 2 marks

- C4** A shop records, for 12 customers in a queue, whether each one pays by card. [2 marks]
People who shop together usually pay the same way. Explain why a binomial distribution may not be a suitable model for the number who pay by card.
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- C5** The heights of one-year-old trees are normally distributed with mean 85 cm [2 marks]
and standard deviation 7 cm. Estimate the percentage of the trees that are taller than 92 cm.

Answer

- C6** The lengths of some bolts are normally distributed with mean 50 mm. 16% of [2 marks]
the bolts are longer than 50.8 mm. Work out the standard deviation.

Answer

- C7** Test scores are normally distributed with mean 65 and standard deviation 8.

- (a) Between which two scores do the middle 68% of the scores lie? [1 mark]

Answer and

- (b) Between which two scores do almost all (99.7%) of the scores lie? [1 mark]

Answer and

Total for C7: 2 marks

- C8** At a company, the numbers of days off sick last year have mean 6 days and [2 marks]
standard deviation 6 days. Explain why a normal distribution is not a good model for these numbers.
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- C9** The masses of bags of pasta are normally distributed with mean 250 g and [2 marks]
standard deviation 5 g. Sketch the distribution, with a scale running from 3 sd below the mean up to 3 sd above.

Answer

D Exam-style questions

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** $X \sim B(10, 0.3)$. Sam says, "The mean of X is 3, so $P(X = 3)$ is more than 0.5." [2 marks]

Is Sam correct? You must show your working.

- D2** Each bulb sold by a garden centre flowers with probability 0.9. The bulbs are sold in packs of 8.

- (a) State one assumption you need to make to use a binomial model for the number of bulbs in a pack that flower. [1 mark]

- (b) Find the probability that all 8 bulbs in a pack flower. Give your answer to 3 decimal places. [1 mark]

Answer

- (c) Find the probability that at least 7 bulbs in a pack flower. Give your answer to 3 decimal places. [2 marks]

Answer

- (d) A customer buys 5 packs. Find the probability that at least 7 bulbs flower in every pack. Give your answer to 3 decimal places. [2 marks]

Answer

Total for D2: 6 marks

- D3** A dentist has 25 appointments each day. Each patient misses the appointment with probability 0.08, independently.

- (a) Work out the mean number of missed appointments in a day. [1 mark]

Answer

- (b) Find the probability that exactly 2 appointments are missed in a day. Give your answer to 3 decimal places. [2 marks]

Answer

- (c) Find the probability that at least one appointment is missed in a day. Give your answer to 3 decimal places. [1 mark]

Answer

Total for D3: 4 marks

D4 Two farms grow tomatoes. The masses of tomatoes from farm P are normally distributed with mean 124 g and standard deviation 8 g. The masses from farm Q are normally distributed with mean 115 g and standard deviation 15 g. A shop only buys tomatoes heavier than 100 g.

- (a) For each farm, estimate the percentage of tomatoes with a mass less than 100 g. [2 marks]

Answer

- (b) Which farm should the shop buy from? Give a reason. [1 mark]

- (c) Compare the masses of the tomatoes from the two farms. [2 marks]

Total for D4: 5 marks

D5 The masses of a type of potato are normally distributed. 16% of the potatoes have a mass less than 345 g and 2.5% have a mass more than 390 g. Find the mean and standard deviation of the masses. [3 marks]

Answer mean g sd g

D6 The masses of 200 parcels have mean 2.4 kg and standard deviation 0.3 kg. 141 of the parcels have a mass between 2.1 kg and 2.7 kg. Does a normal distribution seem a good model for these masses? Give a reason. [3 marks]

D7 A firm checks each batch of its products by testing 10 items. The batch is accepted if at most 1 of the 10 is faulty. 5% of the items are faulty.

- (a) Find the probability that a batch is accepted. Give your answer to 3 decimal places. [3 marks]

Answer

- (b) The firm says, "At least 9 in every 10 batches will be accepted." Use your answer to (a) to comment on this. [1 mark]

Total for D7: 4 marks

D8 For each variable, say whether a binomial or a normal distribution is the better model. Give a reason for each.

- (a) The number of heads when a coin is thrown 20 times. [1 mark]

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- (b) The heights of adult women in a town. [1 mark]

Total for D8: 2 marks

E Extension

No route is given. Spend five minutes on a problem before you look at a hint. The hints are at the end of the sheet. Use one at a time.

- E1** $X \sim B(4, p)$ and $P(X = 0) = 0.0256$. Work out $P(X = 4)$. [3 marks]

Answer

- E2** Ella and Finn each take 2 penalties. Ella scores each one with probability 0.8 and Finn with probability 0.6, all independently. Work out the probability that Ella scores more goals than Finn. [4 marks]

Answer

- E3** The lengths of some screws are normally distributed with mean 40 mm and standard deviation 0.5 mm. A screw is rejected if it is longer than 41 mm. Three screws are chosen at random. Find the probability that at least one of them is rejected. Give your answer to 3 decimal places. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can state the conditions for a binomial model in context and work out $P(X = r)$, at least and at most, and the mean np .	☐	☐	☐
I can use 68%, 95% and 99.7% and symmetry to estimate from a normal distribution, sketch it with its scale, and say when data are not normal.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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|-----------|---------------|--|
| E1 | Hint 1 | $P(X = 0)$ is the probability of 4 failures. |
| | Hint 2 | $(1 - p)^4 = 0.0256$, so $1 - p = 0.4$. |
| | Hint 3 | $p = 0.6$, and $P(X = 4) = p^4$. |
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|-----------|---------------|---|
| E2 | Hint 1 | List the ways Ella can score more: 1 against 0, 2 against 0, 2 against 1. |
| | Hint 2 | Ella: $P(0) = 0.04$, $P(1) = 0.32$, $P(2) = 0.64$. Finn: $P(0) = 0.16$, $P(1) = 0.48$, $P(2) = 0.36$. |
| | Hint 3 | $0.32 \times 0.16 + 0.64 \times (0.16 + 0.48)$. |
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| E3 | Hint 1 | How many standard deviations above the mean is 41 mm? |
| | Hint 2 | 41 is 2 sd above, so $P(\text{rejected}) = 0.025$. |
| | Hint 3 | At least one is $1 - 0.975^3$. |