

Conditional probability

About 55 minutes Calculator allowed 40 marks

Hints online, answers free with an account:
mathswariz.uk/worksheets/stats-conditional-probability/

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

- A1** Needed today [2 marks] **A2** From chapter 8 [2 marks]
- A bag has 6 blue pens and 3 black pens. Two are taken out at random, one after the other, and not put back. Find the probability that they are the same colour.
- Over four years the price of a litre of petrol rose by 11%, rose by 9%, rose by 6% and then fell by 2%. Work out the average percentage increase per year, to 1 decimal place.

Answer

Answer

- A3** From chapter 11 [2 marks] **A4** Maths skill [2 marks]
- The probability that a customer at a café buys a drink is 0.35. The probability that the customer buys a cake is 0.6, and the probability that the customer does both is 0.1. Find the probability that the customer buys a drink or a cake or both.
- Amy and Ben share some sweets in the ratio 3 : 5. Ben gets 8 more sweets than Amy. How many sweets are there altogether?
- Answer

Answer

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$.
Work out $P(A | B)$. Are A and B independent?

Hint $P(A | B) = \frac{P(A \cap B)}{P(B)}$. Independent events have $P(A | B) = P(A)$.

- B3** $P(X) = 0.6$, $P(Y) = 0.35$ and $P(X \cap Y) = 0.24$. Are X and Y independent? You must show your working. [2 marks]
- _____

EXAMPLE 2

The table shows how 150 students have lunch. (a) Find the probability that a Year 11 student, picked at random, brings a packed lunch. (b) Find the probability that a student with a packed lunch, picked at random, is in Year 11.

Lunch	School	Packed
Year 10	48	32
Year 11	30	40

Hint The condition tells you which group you choose from: that group's total is the denominator.

B1 The table shows the main activity of 200 people at a sports club.

Main activity	Swim	Run
Male	34	56
Female	52	58

(a) Find the probability that a female member, picked at random, mainly swims. [1 mark]

Answer

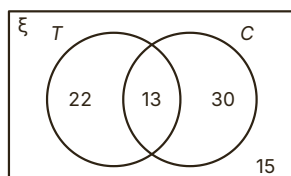
(b) Find the probability that a member who mainly swims, picked at random, is female. [2 marks]

Answer

Total for B1: 3 marks

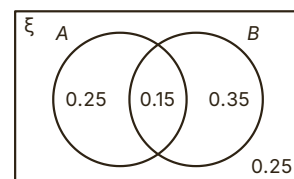
EXAMPLE 3

The Venn diagram shows 80 people and whether they drink tea (T) or coffee (C). Work out (a) $P(C | T)$, (b) $P(C | T')$.



Hint T' is everyone outside circle T , including the people outside both circles.

B2 The Venn diagram gives P for each region, for events A and B .



(a) Work out $P(A | B)$. [1 mark]

Answer

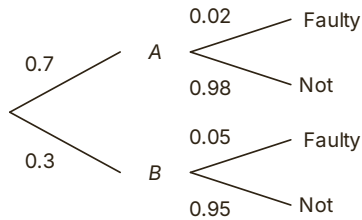
(b) Work out $P(B | A')$. [2 marks]

Answer

Total for B2: 3 marks

EXAMPLE 4

A factory has two machines. Machine A makes 70% of the parts and machine B makes 30%. 2% of the parts from A and 5% of the parts from B are faulty. A part is faulty. Work out the probability that it was made by machine B .



Hint Work out $P(\text{faulty})$ first: it is the denominator.

B4 A shop sends 22% of its parcels by van and the rest by post. 9% of the van parcels and 4% of the posted parcels arrive late.

- (a) Work out the probability [2 marks]
that a parcel arrives late.

Answer

- (b) A parcel arrives late. [2 marks]
Work out the probability
that it was sent by van.

Answer

Total for B4: 4 marks

c Practice

- C1** For two events, $P(A) = 0.35$ and [2 marks]
 $P(A \cap B) = 0.14$. Work out
 $P(B | A)$.

Answer

- C2** The probability that a match is [2 marks]
played in rain is 0.3. Given rain,
the probability that the match is
called off is 0.7. Work out the
probability that a match is
played in rain and is called off.

Answer

- C3** Of 200 students, 120 walk to [2 marks]
school, 50 have a dog and 30
walk to school and have a dog.
Are walking to school and
having a dog independent?
Show how you decide.

- C4** The table shows 1000 men. Liam says, [2 marks]
"The probability that a man with heart
disease smokes is $\frac{30}{200}$."

Men	Heart disease	No heart disease
Smoker	30	170
Non-smoker	20	780

- (a) What probability has Liam [1 mark]
worked out?

- (b) Work out the probability that [2 marks]
a man with heart disease
smokes.

Answer

Total for C4: 3 marks

D Exam-style questions

- D1** For two events, $P(A) = 0.3$ and $P(B) = 0.4$. A and B are mutually exclusive. [2 marks]
Sam says, "Mutually exclusive events are independent."
Use these events to show that Sam is wrong.

- D2** A survey of 200 households found that 46 have a dog but no garden, 64 have a dog and a garden, 38 have a garden but no dog and 52 have neither. [2 marks]
- (a) Find the probability that a household with a dog, picked at random, has a garden.

Answer

- (b) Compare the probability of having a dog for households with a garden and households without a garden. [2 marks]

- (c) Are having a dog and having a garden independent? Give a reason. [2 marks]

Total for D2: 6 marks

E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A bag has 4 red and 6 blue counters. Ella takes two of them at random, one after the other, and does not put the first back. At least one of her counters is red. [3 marks]
Work out the probability that both are red.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** The condition "at least one red" is the denominator.
Hint 2 $P(\text{both red}) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$ and
 $P(\text{at least one red}) = 1 - \frac{6}{10} \times \frac{5}{9} = \frac{60}{90}$.
Hint 3 $\frac{12}{90} \div \frac{60}{90} = \frac{12}{60} = \frac{1}{5}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out a conditional probability from a table, a Venn diagram or a tree, using the group in the condition as the denominator.



I can use $P(B | A) = \frac{P(A \cap B)}{P(A)}$, forwards and backwards.



I can test whether two events are independent and give a conclusion.

