



Histograms, frequency polygons and cumulative frequency

About 85 minutes Calculator allowed 81 marks

Date _____

A Do Now

The first two are what this chapter needs. The rest bring back earlier chapters so nothing is forgotten.

A1 Needed today [2 marks]

Eight times, in seconds: 20.7, 28.2, 22.2, 25.0, 15.8, 26.4, 20.0, 17.9. How many of them are in the class $20 < t \leq 25$?

Answer

A2 Needed today [2 marks]

Given that $43 \times 38 = 1634$, work out $1634 \div 0.38$.

Answer

A3 Maths skill [2 marks]

$M(-3, 4)$ is the midpoint of $A(-5, -4)$ and B . Find the coordinates of B .

Answer

A4 Maths skill [2 marks]

Round 0.8003 to 3 significant figures.

Answer

Write 0.1875 as a fraction in its simplest form.

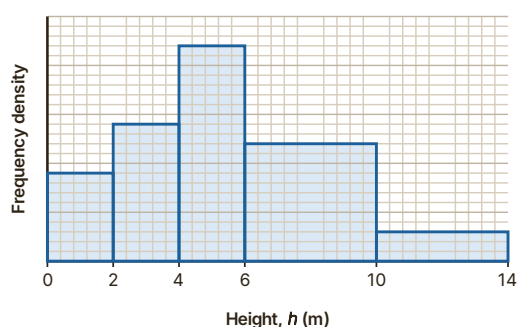
Answer

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

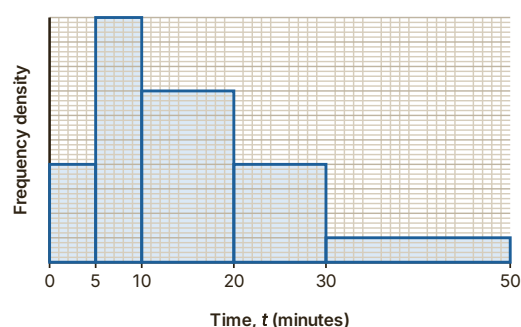
The histogram shows the heights, h m, of 150 trees in a wood. There is no scale on the vertical axis. Estimate how many of the trees are between 4 m and 7 m tall.



Hint Find the total area of the bars. It stands for 150 trees.

B1 [3 marks]

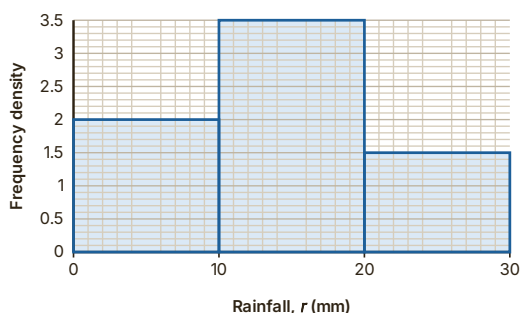
The histogram shows the times, t minutes, that 200 shoppers queued at a post office. There is no scale on the vertical axis. Estimate how many queued for more than 15 minutes.



Answer

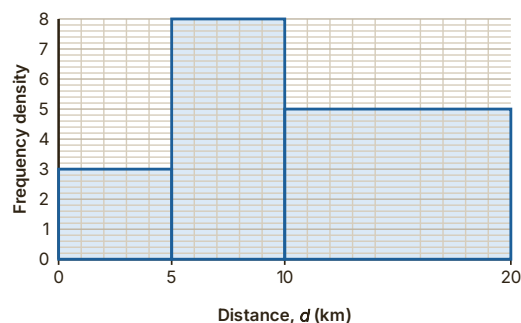
EXAMPLE 2

The histogram shows the rainfall, r mm, on 70 days. On 14 more days the rainfall was in the class $30 < r \leq 50$. Work out the height of the bar for this class.



Hint Height = frequency density = frequency $\div$ class width.

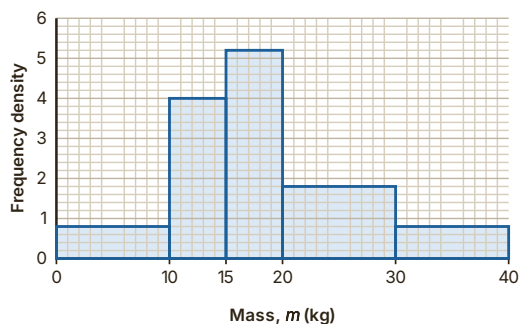
B2 The histogram shows the distances, d km, that 105 people cycled to work. [2 marks]
Another 24 people cycled in the class $20 < d \leq 40$. Work out the height of the bar for this class.



Answer

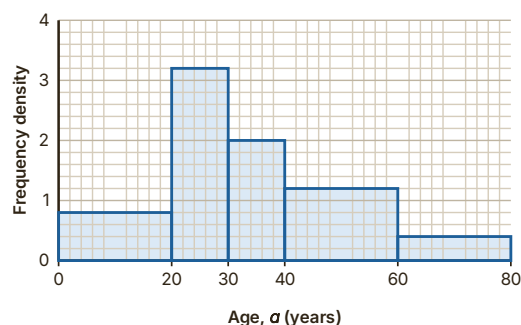
EXAMPLE 3

The histogram shows the masses, m kg, of 80 dogs at a vet's. One dog is chosen at random. Find the probability that it weighs more than 20 kg.



Hint Find the number of dogs over 20 kg first.

B3 The histogram shows the ages, a years, of 100 members of a library. One member is chosen at random. Find the probability that the member is older than 40. [2 marks]



Answer

EXAMPLE 4

The table shows the times, t minutes, of 50 deliveries. List the coordinates you would plot to draw a cumulative frequency graph. Explain why each point is plotted at the upper class boundary.

Time, t (minutes)	Frequency
$0 < t \leq 10$	6
$10 < t \leq 20$	14
$20 < t \leq 30$	18
$30 < t \leq 40$	8
$40 < t \leq 50$	4

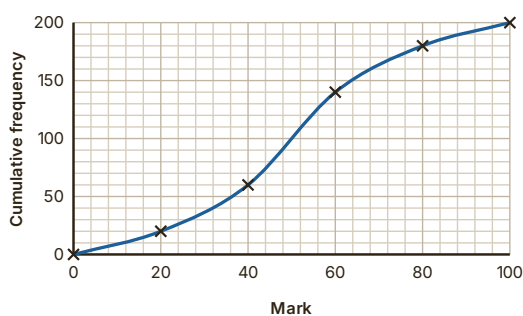
Hint Add the frequencies as you go down.

- B4** The table shows the masses, m kg, of 50 lambs. [3 marks]
Write down the cumulative frequencies. Explain why the point for the class $70 < m \leq 80$ is plotted at 80 and not at 75.

Mass, m (kg)	Frequency
$60 < m \leq 70$	5
$70 < m \leq 80$	12
$80 < m \leq 90$	20
$90 < m \leq 100$	9
$100 < m \leq 110$	4

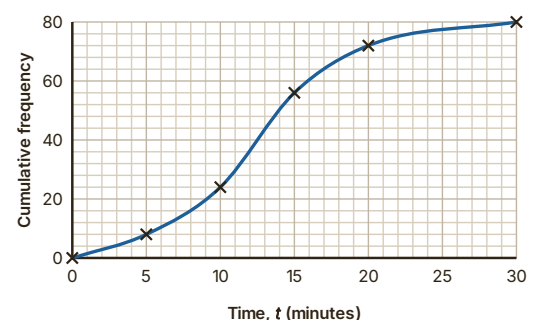
EXAMPLE 5

The cumulative frequency graph shows the marks of 200 students in a test. Work out the interdecile range. Interpret the 90th percentile in context.



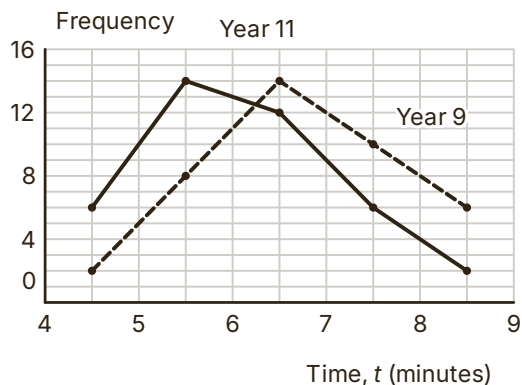
Hint The 10th percentile is the value 10% of the way up the 200 students.

- B5** The cumulative frequency graph shows the times, t minutes, that 80 patients waited at a hospital. Work out the 30th and 70th percentiles. Interpret the 70th percentile in context. [3 marks]



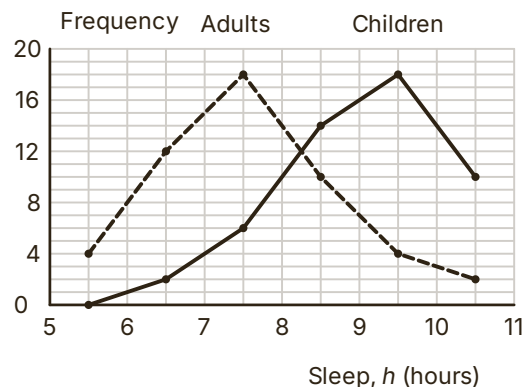
EXAMPLE 6

The frequency polygons show the times, t , minutes, that 40 Year 9 and 40 Year 11 students took to run 1 km. Compare the two distributions.



Hint Compare where each polygon peaks, and how wide each one is.

B6 The frequency polygons show the hours of sleep, h , of 50 children and 50 adults one night. Compare the two distributions. [2 marks]



C Practice

Each question asks for something different.

C1 On a histogram, the bar for $10 < t \leq 35$ has a frequency density of 2.4. The class $35 < t \leq 60$ has a frequency of 40. Which of the two classes has more values? Show how you decide. [2 marks]

Answer

C2 45 parcels have masses in the class $40 < m \leq 70$ grams. Estimate how many of them weigh more than 64 grams. State the assumption you make. [2 marks]

C3 A cumulative frequency graph shows the times of 160 runners. Write down the cumulative frequency you would read across from to find the 35th percentile, and to find the 7th decile. [2 marks]

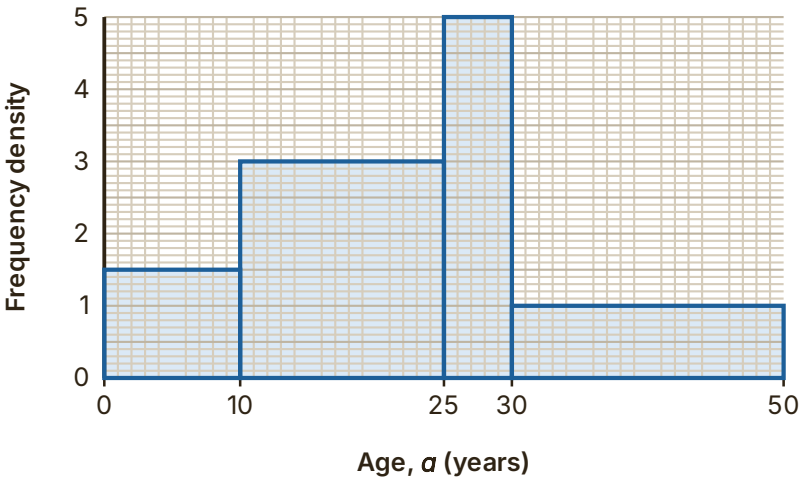
Answer

- C4** The cumulative frequency table shows the masses, w grams, of 100 tomatoes. [2 marks]
Write down the class that contains the median and the class that contains the upper quartile.

Mass, w (grams)	Cumulative frequency
$w \leq 10$	12
$w \leq 20$	40
$w \leq 30$	70
$w \leq 40$	92
$w \leq 50$	100

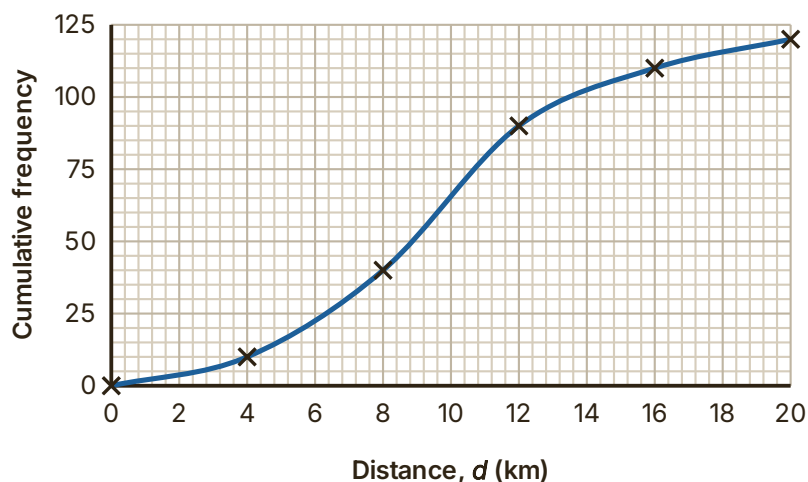
Answer

- C5** The histogram shows the ages, a years, of 105 cyclists in a race. Estimate how many of them are aged between 20 and 40. [3 marks]



Answer

- C6** The cumulative frequency graph shows the lengths, d km, of 120 taxi journeys. [2 marks]
Work out the percentage of the journeys that were longer than 16 km. Give your answer to 1 decimal place.



Answer

- C7** Lily says, "The 40th percentile of the heights is 152 cm, so 40 of the students are 152 cm or shorter." [2 marks]

There are 80 students in Lily's year group. Is Lily correct? Give a reason.

- C8** On a histogram with no vertical scale, a bar with an area of 30 small squares stands for 12 people. How many people does a bar with an area of 45 small squares stand for? [2 marks]

Answer

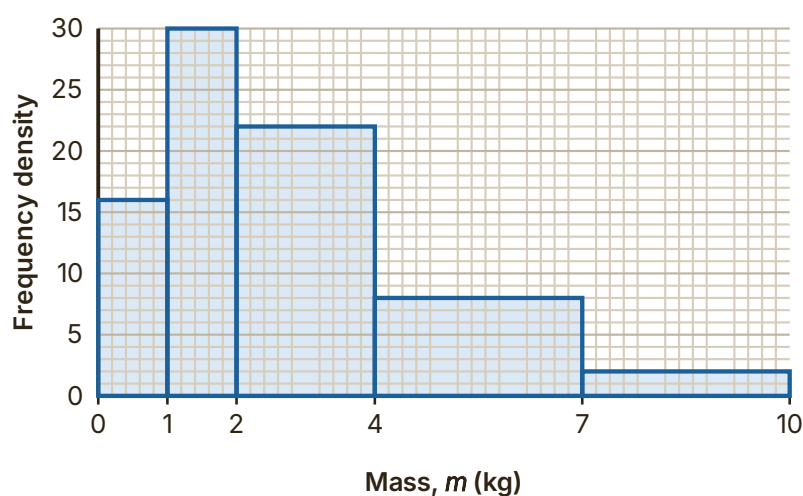
D Exam-style questions

Show your working: most of the marks are for method.

- D1** Priya says, "The 70th percentile of the 200 finishing times is 50 minutes, so 70% of the runners took longer than 50 minutes." [3 marks]

Is Priya correct? Give a reason, and say how many runners took longer than 50 minutes.

- D2** The histogram shows the masses, m kg, of 120 parcels sent by a firm.



- (a) Estimate how many parcels have a mass between 1.5 kg and 3 kg. [2 marks]

Answer

- (b) A parcel is chosen at random. Find the probability that its mass is more than 4 kg. [2 marks]

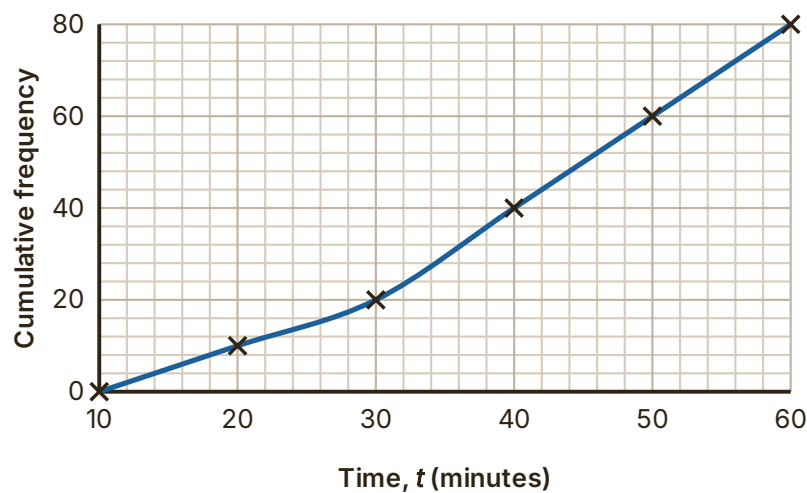
Answer

- (c) The firm charges extra for a parcel over 5 kg. Estimate the percentage of these parcels that are charged extra. Give your answer to 1 decimal place. [2 marks]

Answer

Total for D2: 6 marks

D3 The cumulative frequency graph shows the times, t minutes, that 80 Year 10 students spent on homework one evening.



(a) Use the graph to estimate the median time. [1 mark]

Answer

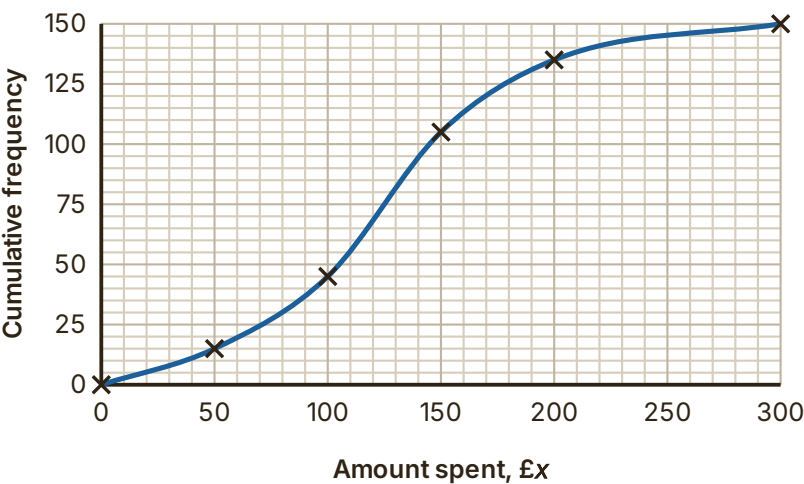
(b) Use the graph to estimate the interquartile range. [2 marks]

Answer

(c) For 80 Year 8 students, the median time was 36 minutes and the interquartile range was 12 minutes. Compare the times of the two year groups. [2 marks]

Total for D3: 5 marks

D4 The cumulative frequency graph shows the amounts, £ x , that 150 shoppers spent in a store one Saturday.



(a) Work out the 90th percentile. Interpret it in context. [2 marks]

(b) Work out the number of shoppers who spent more than £100 but no more than £200. [2 marks]

Answer _____

Total for D4: 4 marks

D5 The table shows the ages, a years, of 120 people at a concert. Estimate how many of them are aged between 25 and 50. Is your answer likely to be exact? Give a reason. [4 marks]

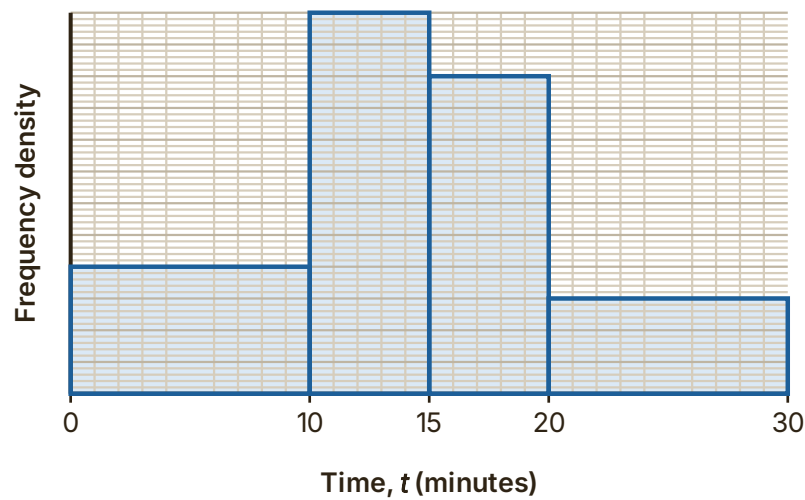
Age, a (years)	Frequency
$0 < a \leq 10$	12
$10 < a \leq 30$	48
$30 < a \leq 40$	24
$40 < a \leq 70$	36

D6 On a histogram of car speeds, s mph, 1 cm^2 stands for 4 cars and the horizontal scale is 1 cm to 5 mph. The class $50 < s \leq 60$ has 36 cars. Work out the height of its bar, in cm. [3 marks]

Answer

D7 A sports scientist has the times of 200 runners in a grouped frequency table. [2 marks]
She wants to estimate the interquartile range. Should she draw a histogram or a cumulative frequency graph? Give a reason.

D8 The histogram shows the times, t minutes, that 90 patients waited to see a nurse. There is no scale on the vertical axis. A patient is chosen at random. Find the probability that the patient waited more than 15 minutes. [3 marks]

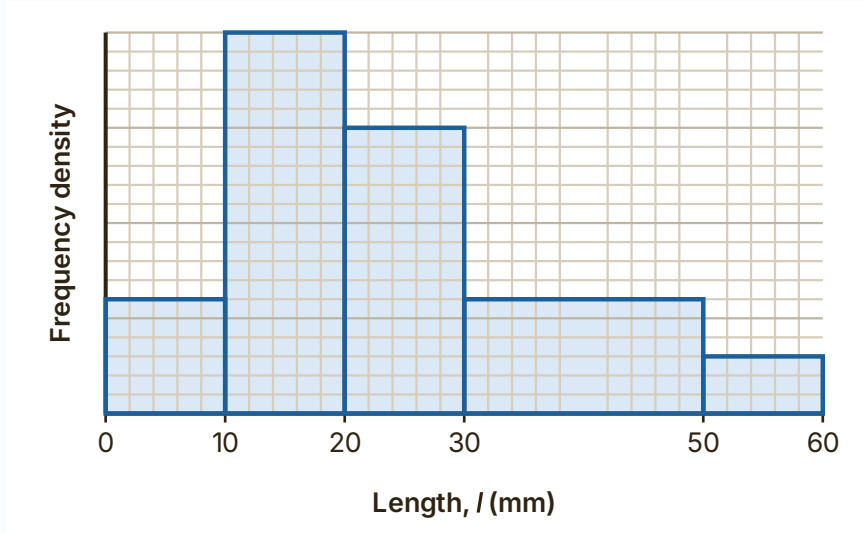


Answer

E Extension

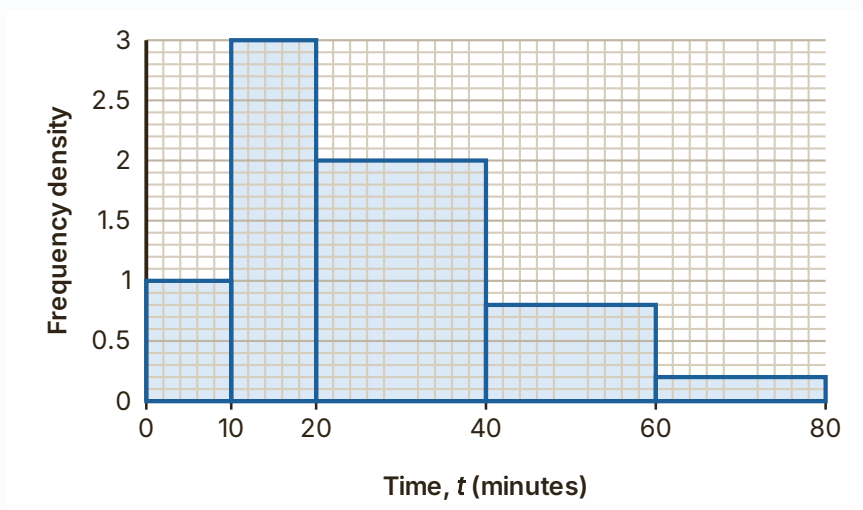
No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The histogram shows the lengths, l mm, of some insects. It has no vertical scale. There are 16 more insects in the class $10 < l \leq 20$ than in the class $30 < l \leq 50$. How many insects are there altogether? [3 marks]



Answer

- E2** The histogram shows the times, t minutes, that 100 people spent at a gym. Estimate the interquartile range. [3 marks]



Answer

E3 Kemi sat two tests. In Test A, her mark was 74 and the cumulative frequency at 74 was 168 out of 240 students. In Test B, her mark was 82 and the cumulative frequency at 82 was 231 out of 300 students. In which test did Kemi do better compared with the other students? Give a reason.

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can read and complete histograms with or without a scale, and use frequency polygons to compare.	☐	☐	☐
I can draw and read cumulative frequency graphs, and find and interpret percentiles.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	Each bar's area stands for its frequency. Write both areas in small squares.
	Hint 2	The difference between the two areas stands for 16 insects.
	Hint 3	Find what one small square stands for, then use the total area.

E2	Hint 1	Find the frequency of each bar, then the running totals.
	Hint 2	The lower quartile is the 25th person: they are in $10 < t \leq 20$, 15 people into a class of 30.
	Hint 3	The upper quartile is the 75th person, 35 people into the 40 in $20 < t \leq 40$.

E3	Hint 1	Turn each cumulative frequency into a percentage of its group.
	Hint 2	That percentage is the percentile of Kemi's mark.
	Hint 3	The higher percentile is the better result compared with the group.