



Index numbers, rates and control charts

About 85 minutes

Calculator allowed 86 marks

Date _____

A Do Now

The first two are what this chapter needs. The rest bring back earlier chapters so nothing is forgotten.

A1 Needed today [2 marks]

After a 5% increase, the price of a bike is £252. What was the price before the increase?

Answer

A2 Needed today [2 marks]

Write $\frac{3}{5} : \frac{2}{3}$ as a ratio of whole numbers in its simplest form.

Answer

A3 Maths skill [2 marks]

Given that $\frac{6}{36} = \frac{5}{x}$, find x .

Answer

A4 Maths skill

[2 marks]

The point $(4, 0)$ is reflected in the line $x = 2$. Find the coordinates of its image.

Answer

A5 Maths skill

[2 marks]

$\frac{2}{3}$ of a number is 10. What is the number?

Answer

B Examples, then your turn

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

The table shows the index numbers and weights for three ingredients a school canteen buys. Work out the weighted index number.

Ingredient	Flour	Cheese	Oil
Index number	112	104	120
Weight	5	3	2

Hint Multiply each index number by its weight, add, then divide by the total weight.

B1 The table shows the index numbers and weights for a family's costs. Work out the weighted index number. [2 marks]

Cost	Rent	Food	Energy	Travel
Index number	109	115	103	98
Weight	30	25	35	10

Answer

EXAMPLE 2

The chain base index numbers for a bus fare in three years were 103, 106 and 104. Work out the average yearly rate of change. Give your answer to 1 decimal place and interpret it.

Hint Multiply the three as decimals, then take the cube root.

B2 A phone's price had chain base index numbers of 98, 101, 95 and 99 over four months. Work out the average monthly rate of change. Give your answer to 1 decimal place and interpret it. [3 marks]

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EXAMPLE 3

A town's death rates are 3, 9 and 60 per 1000 for people aged under 40, 40 to 69, and 70 and over. The standard population is 25%, 55% and 20% in these groups. Work out the standardised death rate.

Hint Weight each rate by the standard population's percentage.

B3 A district's death rates are 1.4, 6 and 52 per 1000 for people aged under 40, 40 to 69, and 70 and over. The standard population is 30%, 50% and 20% in these groups. Work out the standardised death rate. Give your answer to 1 decimal place. [3 marks]

Answer

EXAMPLE 4

A town has 64 000 people, of whom 13 500 are women aged 15 to 44. There were 945 births last year. Work out the crude birth rate and the general fertility rate.

Hint The crude rate divides by everyone; the general fertility rate divides by the women aged 15 to 44 only.

B4 A city has 82 000 people, of whom 17 600 are women aged 15 to 44. There were 1144 births last year. Work out the crude birth rate and the general fertility rate. [3 marks]

Answer

EXAMPLE 5

Jars of jam have a target mass of 750 g, and the sample means have a standard deviation of 3 g. A sample of four jars has masses 754, 761, 757 and 760 g. What should be done?

Hint Work out the limits first, then the sample mean.

B5 Bottles of water have a target of 2000 ml, and the sample means have a standard deviation of 8 ml. A sample of four bottles has volumes 2023, 2031, 2026 and 2028 ml. What should be done? [3 marks]

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EXAMPLE 6

One sample mean on a control chart is between the upper warning limit and the upper action limit. Explain why the process is not stopped straight away.

Hint How often is a mean outside the warning limits when nothing is wrong?

B6 A sample mean on a control chart is above the upper action limit. Explain why the process should be stopped and adjusted. [2 marks]

c Practice

Each question asks for something different. Show your working.

C1 A loaf of bread cost £3.60 in 2022 and £4.14 in 2025. Work out the index number for 2025 with 2022 as the base year, and say what it means. [2 marks]

C2 A bike cost £350 in 2020. Its index number for 2025, with 2020 as the base year, is 92. Work out the price of the bike in 2025. [2 marks]

Answer _____

C3 The chain base index numbers for a cinema ticket in three years were 104, 105 and 110. Work out the percentage increase in the price over the three years. [2 marks]

Answer _____

C4 There were 1248 deaths last year in a city with a population of 96 000. What was its crude death rate? [2 marks]

Answer _____

C5 A county's general fertility rate is 58 per 1000, and it has 21 500 women aged 15 to 44. How many births were there? [2 marks]

Answer _____

C6 A control chart for the lengths of nails has warning limits of 37.2 mm and 42.8 mm. Work out the target length and the standard deviation of the sample means. [2 marks]

Answer

C7 The range chart for bags of rice has an upper warning limit of 10 g and an upper action limit of 14 g. A sample of five bags has masses 48, 57, 53, 61 and 50 g (above 1 kg). What should be done? Give a reason. [2 marks]

C8 A process is working correctly and 400 sample means are plotted. About how many would you expect to be outside the action limits? [2 marks]

Answer

D Exam-style questions

Answer every question in the space given. Show your working: most of the marks are for method.

D1 The index numbers for a train season ticket, with 2020 as the base year, are 110 in 2023 and 121 in 2024. Amy says, "The price rose by 11% from 2023 to 2024." [2 marks]

Is Amy correct? You must show your working.

D2 The table shows the index numbers and weights for the costs of running a car.

Cost	Fuel	Insurance	Servicing	Tax
Index number	118	108	104	102
Weight	40	30	20	10

(a) Work out the weighted index number for the cost of running the car. [2 marks]

Answer

(b) Over the same time the Consumer Price Index rose from 100 to 106. Compare the cost of running the car with general prices. [2 marks]

- (c) Give one reason why this index may not show the change in costs for one particular driver. [1 mark]

Total for D2: 5 marks

D3 The table shows the chain base index numbers for the mean price of a house in a village. In 2020 the mean price was £180 000.

Year	2021	2022	2023	2024
Chain base index	108	106	97	104

- (a) Work out the mean price in 2024. Give your answer to the nearest pound. [2 marks]

Answer

- (b) Work out the average yearly rate of change in the price. Interpret your answer. [2 marks]

Total for D3: 4 marks

D4 The table shows the people and the deaths in one year in two towns, P and Q, and a standard population.

Age (years)	Under 40	40 to 69	70 and over
Town P people	30 000	25 000	15 000
Town P deaths	30	150	900
Town Q people	40 000	20 000	5 000
Town Q deaths	48	140	325
Standard population	50%	35%	15%

- (a) Work out the crude death rate for town P. Give your answer to 1 decimal place. [1 mark]

Answer

- (b) Use the standard population to find each town's standardised death rate. [3 marks]

Answer

- (c)

The crude death rate for town Q is 7.9 per 1000. Use your answers to compare the two towns.

[2 marks]
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Total for D4: 6 marks

D5 The table gives facts about two regions, R and S.

Region	R	S
Population	120 000	80 000
Women aged 15 to 44	22 000	20 000
Births in a year	1430	1240

- (a)

Work out the general fertility rate for each region.

[2 marks]

Answer

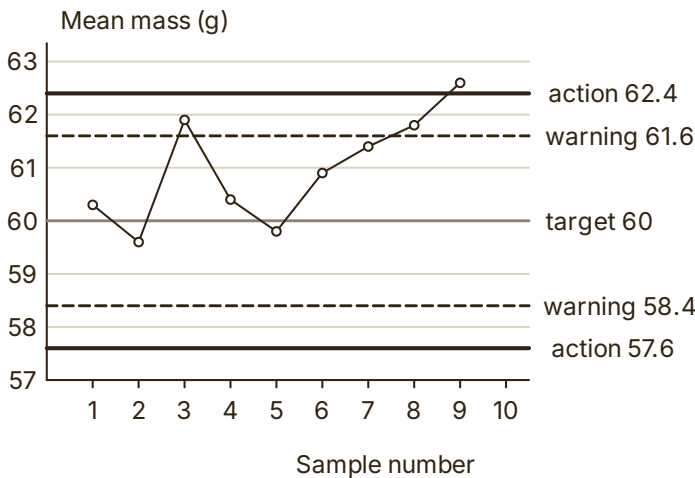
- (b)

The crude birth rate is 11.9 for R and 15.5 for S. Which region has the higher fertility? Explain why the crude birth rates give a different answer.

[2 marks]
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Total for D5: 4 marks

D6 A machine fills bags of nuts with a target mean of 60 g. The control chart shows the sample means for the first 9 samples.



- (a)

What should have been done after sample 3?

[1 mark]
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(b) What should be done after sample 9? Give a reason. [1 mark]

(c) Describe the pattern in samples 5 to 9, and say what it shows about the machine. [2 marks]

Total for D6: 4 marks

D7 Cans of drink have a target volume of 330 ml. The standard deviation of the sample means is 1.5 ml.

(a) Find the two warning limits and the two action limits. [2 marks]

Answer

(b) A sample of five cans has volumes 333.1, 334.0, 332.6, 333.8 and 335.0 ml. [2 marks]
What should be done? You must give a reason.

Total for D7: 4 marks

D8 The Consumer Price Index rose from 128.4 to 133.5 over one year. Leon's pension rose by 3% over the same year. Is Leon's pension keeping up with prices? You must show your working. [3 marks]

E Extension

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 A gym membership has an index number of 108 in 2024, using 2022 as the base year. The chain base index number for 2024 is 104. Work out the chain base index number for 2023. Give your answer to 1 decimal place. [3 marks]

Answer

E2 A town's death rates are 2, 10 and 70 per 1000 for people aged under 40, 40 to 69, and 70 and over. Its standardised death rate is 13.6 per 1000. The standard population has 30% aged under 40. What percentage is aged 40 to 69? [3 marks]

Answer

E3 A control chart for boxes of cereal has warning limits of 81.2 g and 88.8 g (above 500 g). A sample mean is 89.9 g. What should be done? You must show your working. [3 marks]

E4 The chain base index numbers for a price over four years have a geometric mean of 104. The first three are 102, 106 and 103. Work out the fourth. Give your answer to 1 decimal place. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can work out simple, chain base and weighted index numbers, and an average rate with the geometric mean.	☐	☐	☐
I can work out crude and standardised rates and the general fertility rate, and explain why crude rates mislead.	☐	☐	☐
I can work out warning and action limits and decide what to do after a sample.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	The chain base numbers for 2023 and 2024 multiply to give the change from 2022 to 2024.
	Hint 2	So $(\text{chain 2023}) \times 1.04 = 1.08$.
	Hint 3	$\text{Chain 2023} = 1.08 \div 1.04 \times 100$.

E2	Hint 1	Call the percentage aged 40 to 69 x . Then $70 - x$ per cent are aged 70 and over.
	Hint 2	$2 \times 0.3 + 10 \times \frac{x}{100} + 70 \times \frac{70-x}{100} = 13.6$.
	Hint 3	This gives $49.6 - 0.6x = 13.6$.

E3	Hint 1	The target is halfway between the warning limits, and each is 2 standard deviations from it.
	Hint 2	The target is 85 and the standard deviation is 1.9, so the action limits are 85 ± 5.7 .
	Hint 3	Compare 89.9 with 88.8 and with 90.7.

E4	Hint 1	The product of the four, as decimals, is 1.04^4 .
	Hint 2	$1.04^4 = 1.16985856$ and $1.02 \times 1.06 \times 1.03 = 1.113636$.
	Hint 3	Divide one by the other.