

Probability, risk, Venn and tree diagrams and conditional probability



Hints online, answers free with an account:
mathswariz.uk/worksheets/stats-probability-risk-venn-and-tree-diagrams-and-conditional-probability/

About 85 minutes Calculator allowed 95 marks

Date _____

A Do Now

The first two are what this chapter needs. The rest bring back earlier chapters so nothing is forgotten.

A1 Needed today [2 marks]

A bag holds only red, blue and green counters. $P(\text{red}) = 0.45$ and $P(\text{blue}) = 0.1$. There are 220 counters in the bag. How many are green?

Answer

A2 Needed today [2 marks]

Write $\frac{216}{30}$ as a mixed number in its simplest form.

Answer

A3 From chapter 2 [2 marks]

A researcher wants a sample of 120 of the drivers who use a motorway. Give one advantage of using a quota sample rather than a stratified random sample here.

A4 From chapter 3 [2 marks]

To ask "Have you ever pretended to be ill to miss work?", Noor uses the random response method. Each person thinks of a whole number from 1 to 10: if it is 1, 2 or 3, tick Yes; otherwise answer truthfully. Is this method suitable? Give a reason.

A5 From chapter 3 [2 marks]

A scientist grows bread mould in sealed boxes in a laboratory at four different temperatures. Noor says it is an experiment. Is it a laboratory, field or natural experiment? Give one disadvantage of this type of experiment.

B Examples, then your turn

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

A coin is thrown 500 times and lands heads 340 times. (a) Estimate the probability that the coin lands heads. (b) The coin is thrown 75 more times. Estimate the number of heads.

Hint Relative frequency = $\frac{\text{number of heads}}{\text{number of throws}}$

B1 A factory tests 600 batteries and finds that 27 of them are faulty.

- (a) Estimate the probability [1 mark]
that a battery is faulty.

Answer

- (b) A shop orders 3200 of [2 marks]
these batteries.
Estimate the number of
them that are faulty.

Answer

Total for B1: 3 marks

EXAMPLE 2

A study followed 3000 cyclists who wear a helmet and 750 who do not. 24 of the helmet wearers and 18 of the others had a head injury. For cyclists without a helmet, against those who wear one, find the relative risk of a head injury and say what it means.

Hint Work out the absolute risk for each group first.

B2 A firm has 1600 office workers. 400 of them sit for more than 8 hours a day, and 56 of these have back pain. Of the other 1200 workers, 84 have back pain.

- (a) Find the absolute risk [2 marks] of back pain in each group.

Answer

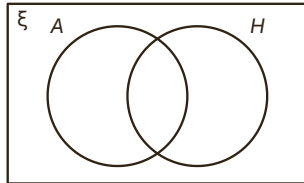
- (b) Find the relative risk of [2 marks] back pain for those who sit for more than 8 hours, against the others. Interpret your answer in context.

.....
.....

Total for B2: 4 marks

EXAMPLE 3

100 visitors to a museum were asked whether they saw the art rooms (A) and the history rooms (H). 62 saw the art rooms, 45 saw the history rooms and 18 saw neither. (a) Fill in the Venn diagram. (b) Find $P(A \cup H)$.



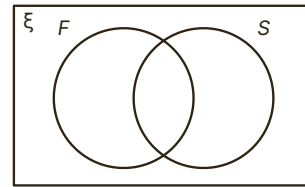
Hint Both = $62 + 45 + 18 - 100$.

EXAMPLE 4

A drawer has 6 white socks and 4 black socks. Two socks are picked out at random, and the first is not put back. Find the probability that they are different colours.

Hint White then black, or black then white: two orders.

B3 80 students were asked whether they study French (F) and whether they study Spanish (S). 44 study French, 37 study Spanish and 13 study neither.



(a) Complete the Venn diagram. [2 marks]

Answer

(b) A student is chosen at random. Work out $P(S \cap F')$. [1 mark]

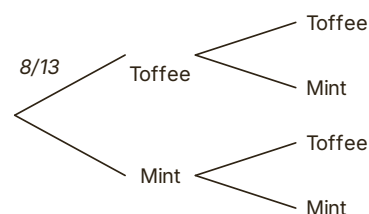
Answer

(c) Work out $P((F \cup S)')$. [1 mark]

Answer

Total for B3: 4 marks

B4 A tin has 8 toffees and 5 mints. Rana picks out two sweets at random, one after the other, keeping the first. Find the probability that she gets two sweets of the same type. You may use the tree diagram. [3 marks]



Answer

EXAMPLE 5

At a garage, 40% of the cars tested are diesel and the rest are petrol. 15% of the diesel cars and 5% of the petrol cars fail the emissions test. A car fails. Work out the probability that it is diesel.

Hint Work out $P(\text{fail})$ from both routes, then divide.

EXAMPLE 6

For a student at a school, the probability of being late is 0.1. On a day when it rains, the probability of being late is 0.25. Are being late and rain independent? Explain what your answer means in context.

Hint Compare $P(\text{late} \mid \text{rain})$ with $P(\text{late})$.

B5 On an airline, 25% of the passengers fly business class and the rest fly economy. 60% of business passengers check in a bag and 30% of economy passengers do. A passenger checks in a bag. Work out the probability that this passenger flies business class. [3 marks]

Answer

B6 At a gym, the probability that a member uses the pool is 0.4. For members over 60, the probability that a member uses the pool is also 0.4. Are using the pool and being over 60 independent? Explain what your answer means in context. [2 marks]

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C Practice

Each question asks for something different. Show your working.

C1 A spinner lands on red, blue, green or yellow. $P(\text{red}) = 0.32$ and $P(\text{blue}) = 0.23$. The probability of green is twice the probability of yellow.

(a) Work out the probability of yellow. [2 marks]

Answer

(b) The spinner is spun 200 times. Estimate the number of times it lands on green. [1 mark]

Answer

Total for C1: 3 marks

- C2** For two events, $P(A) = 0.55$, $P(B) = 0.35$ and $P(A \cap B) = 0.12$. Work out $P(A' \cap B')$. [2 marks]

Answer

- C3** $\xi = \{\text{whole numbers from 1 to 15}\}$. A is the set of multiples of 3 and B is the set of odd numbers.

- (a) List the members of $A \cap B$. [1 mark]

Answer {.....}

- (b) A number is chosen at random from ξ . Work out $P(A \cup B)$. [2 marks]

Answer

Total for C3: 3 marks

- C4** A basketball player scores a free throw with probability 0.85, independently of other throws. She takes two free throws. Work out the probability that she scores exactly one of them. [2 marks]

Answer

- C5** A printer jams on 5% of days, independently of other days. Find the probability that it jams on one or more of three days. Give your answer to 3 decimal places. [2 marks]

Answer

- C6** The table shows the drink chosen by 120 people at a café. One of the coffee drinkers is chosen at random. What is the probability that they are under 40? [2 marks]

Age	Tea	Coffee
Under 40	18	42
40 and over	35	25

Answer

- C7** For two events, $P(A) = 0.45$, $P(B) = 0.4$ and $P(A \cup B) = 0.67$. Are A and B independent? Show your method. [3 marks]

- C8** A game at a fair costs £2 to play. A player rolls two fair dice. A double wins £8; anything else wins nothing. The game is played 360 times. Work out the profit the organiser can expect to make. [3 marks]

Answer

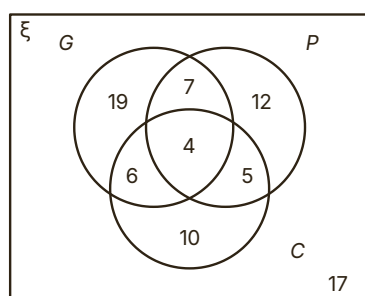
D Exam-style questions

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** A fair coin lands heads on each of its last 5 throws. Priya says, "On the next throw, tails is more likely than heads." [2 marks]

Is Priya correct? Give a reason for your answer.

- D2** The Venn diagram shows the 80 members of a sports centre and whether they use the gym (G), the pool (P) or the courts (C). A member is picked at random.



- (a) Find the probability that the member uses exactly two of the three. [2 marks]

Answer

- (b) The member uses the gym. Work out the probability that the member also uses the courts. [2 marks]

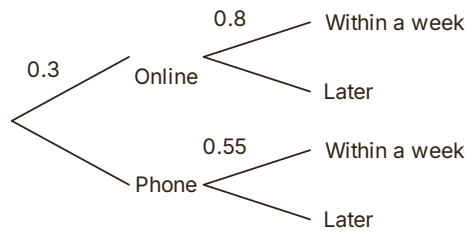
Answer

- (c) Work out the probability that the member uses none of the three. [1 mark]

Answer

Total for D2: 5 marks

- D3** At a doctor's surgery, 30% of the patients book an appointment online and the rest book by phone. 80% of the online bookings are for an appointment within a week. 55% of the phone bookings are for an appointment within a week.



- (a) Fill in the tree diagram. [2 marks]

Answer

- (b) Find the probability that a patient's appointment is within a week. [2 marks]

Answer

- (c) A patient's appointment is within a week. Work out the probability that the patient booked online. [2 marks]

Answer

Total for D3: 6 marks

- D4** A box has 5 red, 4 blue and 3 green pens. Three pens are picked out at random, one at a time, and none is put back. Find the probability that the three pens are all different colours. [4 marks]

Answer

- D5** For two events A and B , $P(A) = 0.5$, $P(B) = x$ and $P(A \cap B) = 0.15$.
 $P(A' \cap B') = 0.25$.

- (a) Work out x . [2 marks]

Answer

- (b) Are A and B independent? You must give a reason. [2 marks]

Total for D5: 4 marks

D6 A study followed 5000 adults for 10 years. 1200 of them did not exercise, and 96 of these developed heart disease. Of the 3800 who did exercise, 114 developed heart disease.

(a) Work out the absolute risk of heart disease for each group. [2 marks]

Answer

(b) Work out the relative risk of heart disease for adults who do not exercise, compared with adults who do. Give your answer to 2 decimal places. [1 mark]

Answer

(c) Interpret your answer to (b) in context. [1 mark]

(d) A town has 12 500 adults who do not exercise. Estimate how many of them will develop heart disease in the next 10 years. [2 marks]

Answer

Total for D6: 6 marks

D7 The table shows 250 drivers, the number of lessons they had and whether they passed their test first time. Is passing first time independent of having more than 30 lessons? You must show your working. [4 marks]

Lessons	Passed first time	Did not pass first time
30 or fewer	63	42
More than 30	87	58

D8 Two fair dice are rolled. At least one of the dice shows a 5. Work out the probability that the total of the two dice is 8. [3 marks]

Answer

D9 A fair dice is rolled. A is the event "an even number", B is "a number greater than 4" and C is "a 1".

(a) Name two of the events that are mutually exclusive, with a reason. [2 marks]

(b) Are A , B and C exhaustive? Explain why.

[1 mark]

Total for D9: 3 marks

E Extension

No route is given. Spend five minutes on a problem before you look at a hint. The hints are at the end of the sheet. Use one at a time.

- E1** A bag has 3 red and 5 blue counters. Two are picked out at random, one after the other, and the first is not replaced. The two counters are the same colour. Work out the probability that they are both red. [3 marks]

Answer

- E2** Events A and B are independent. $P(A) = 0.3$ and $P(A \cup B) = 0.58$. Work out $P(B)$. [3 marks]

Answer

- E3** A seed germinates with probability 0.35, independently of other seeds. What is the least number of seeds that must be planted so that the probability that at least one germinates is greater than 0.95? [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can estimate a probability from relative frequency, find an expected frequency, and work out and interpret absolute and relative risk.	☐	☐	☐
I can complete a Venn diagram, including the region outside both sets, and use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.	☐	☐	☐
I can use tree diagrams for independent events and for selecting without replacement, counting outcomes in every order.	☐	☐	☐
I can find a conditional probability with the right denominator and test whether two events are independent.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- | | | |
|-----------|---------------|--|
| E1 | Hint 1 | This is a conditional probability: the "given" is that the colours are the same. |
| | Hint 2 | $P(\text{both red}) = \frac{3}{8} \times \frac{2}{7}$ and $P(\text{both blue}) = \frac{5}{8} \times \frac{4}{7}$. |
| | Hint 3 | Divide $P(\text{both red})$ by $P(\text{same colour})$. |
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- | | | |
|-----------|---------------|--|
| E2 | Hint 1 | Write $P(B) = p$. What is $P(A \cap B)$ for independent events? |
| | Hint 2 | $0.3 + p - 0.3p = 0.58$. |
| | Hint 3 | $0.7p = 0.28$, so divide by 0.7. |
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- | | | |
|-----------|---------------|--|
| E3 | Hint 1 | "At least one" is everything except "none". |
| | Hint 2 | $P(\text{none}) = 0.65^n$, and this must be less than 0.05. |
| | Hint 3 | Try $n = 5, 6, 7, \dots$ and stop at the first n that works. |