

Quartiles, box plots, standard deviation, skew and standardised scores



Hints online, answers free with an account:
mathswariz.uk/worksheets/stats-quartiles-box-plots-standard-deviation-skew-and-standardised-scores/

About 90 minutes Calculator allowed 86 marks

Date _____

A Do Now

The first two are what this chapter needs. The rest bring back earlier chapters so nothing is forgotten.

A1 Needed today [2 marks]

Over 4 months, the number of visitors to a park rose by 2%, rose by 9%, fell by 8% and rose by 7%. Use the geometric mean to work out the average percentage change per month. Give your answer to 2 decimal places.

Answer

A2 Needed today [2 marks]

$x^2 = 25$ and $y^3 = -8$. Find the smallest possible value of $x + y$.

Answer

A3 From chapter 3 [2 marks]

Pia wants to find out how much people in a city spend on clothes. Pia asks 6 people chosen at random from all the people in the city and says, "My sample is small, so my results are not reliable." Is Pia correct? Give a reason.

Of 50 patients from the morning clinic, 10 waited over an hour. Of 70 patients from the afternoon clinic, 29 waited over an hour. One of the patients who waited over an hour is picked at random. Work out the probability that they are from the afternoon clinic.

Answer

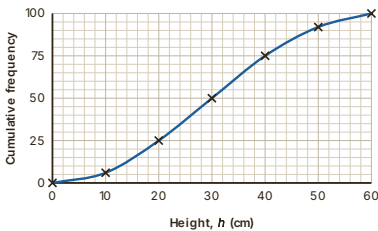
Comparative pie charts show 360 voters in 2023 and 720 in 2024. The sector for those who voted for Party A is 95° in 2023 and 55° in 2024. Femi says fewer voters voted for Party A in 2024. Is Femi correct?

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

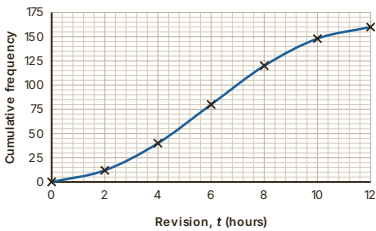
EXAMPLE 1

The cumulative frequency graph shows the heights of 100 plants. Find the median and the interquartile range.



Hint For 100 plants, read across at 25, 50 and 75.

B1 The cumulative frequency graph shows the hours of revision done by 160 students in a week. Find the median and the interquartile range. [3 marks]



Answer Median h IQR h

EXAMPLE 2

The prices of some jackets have a lower quartile of £18 and an upper quartile of £30. The cheapest costs £3 and the dearest costs £52. Show which of these prices are outliers, using the $1.5 \times \text{IQR}$ rule.

Hint Work out $1.5 \times \text{IQR}$, then the lower and upper limits.

- B2** The masses of the dogs at a rescue centre have a lower quartile of 41 kg and an upper quartile of 49 kg. The lightest dog is 28 kg and the heaviest is 60 kg. Using the $1.5 \times \text{IQR}$ rule, which of these masses is an outlier? Show your working. [3 marks]

Answer Outlier:

EXAMPLE 3

The table shows the shoe sizes of 20 children. Work out the mean and the standard deviation.

Shoe size	Frequency
4	3
5	7
6	6
7	4

Hint Work out $\sum fx$ and $\sum fx^2$. Then use $\sqrt{\frac{\sum fx^2}{n} - \bar{x}^2}$.

- B3** The table shows the numbers of goals scored by a team in 20 matches. Work out the mean and the standard deviation. [4 marks]

Goals	Frequency
0	4
1	6
2	7
3	2
4	1

Answer Mean s.d.

EXAMPLE 4

Some test marks out of 40 have a mean of 26 and a standard deviation of 5. Each mark is changed to a percentage by multiplying by 2.5. Find the mean and the standard deviation of the percentages.

Hint Multiplying every value by 2.5 multiplies the mean and the spread by 2.5.

- B4** The midday temperatures in a city have a mean of 68 °F and a standard deviation of 9 °F. Each temperature is changed to °C by taking away 32 and then multiplying by $\frac{5}{9}$. Find the mean and the standard deviation in °C. [3 marks]

Answer Mean °C s.d. °C

EXAMPLE 5

The waiting times at a bank have a mean of 14.2 minutes, a median of 11.5 minutes and a standard deviation of 6 minutes. Work out the skewness, $\frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$, and say what it tells you about the waiting times.

Hint The sign of the skewness gives the direction of the skew. Then say what that means for the waits.

B5 The ages of the people living in a retirement village have a mean of 78.3 years, a median of 80.7 years and a standard deviation of 7.2 years. Work out the skewness and say what it tells you about the ages. [3 marks]

EXAMPLE 6

In golf a lower score is better. Ana scored 68 in a round where the mean was 72 and the standard deviation was 2.5. Ben scored 70 in a round where the mean was 75 and the standard deviation was 4. Who did better, compared with the other players in their round?

Hint Work out each standardised score. Then think about which direction is better in golf.

B6 Kai threw the shot put 12.6 m in Year 10, where the mean was 11.4 m and the standard deviation was 0.8 m. Ola threw 13.8 m in Year 11, where the mean was 12.6 m and the standard deviation was 1.0 m. Who did better, compared with the others in their year? Show your working. [3 marks]

Answer

c Practice

The questions come from all five lessons, in a mixed order.

C1 Here are the ages of 15 people at a party, in order. [2 marks]
21, 24, 25, 27, 30, 31, 33, 35, 36, 40, 42, 45, 50, 58, 63
Work out the interquartile range.

Answer

C2 Here are the numbers of emails 9 people received in an hour. [2 marks]

2, 3, 3, 3, 4, 5, 6, 8, 11

Use the mean, median and mode to describe the skew.

C3 For 12 values, $\sum x = 180$ and $\sum x^2 = 2808$. Work out the standard deviation. [2 marks]

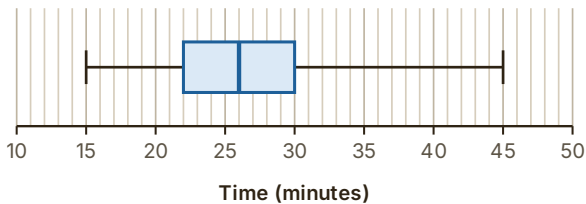
Answer

C4 In a test with a mean of 30 and a standard deviation of 8, Ruby's standardised score was -0.75 . Work out her mark. [2 marks]

Answer

C5 The box plot shows the times, in minutes, that some people took to walk to work. [2 marks]

Is the longest time an outlier? Use the $1.5 \times \text{IQR}$ rule and show your working.



Answer

C6 A set of data has a mean of 40, a standard deviation of 6 and a skewness of 0.5. Work out the median. [2 marks]

Answer

C7 A coach must choose one runner for a relay. Over the season, Priya's 400 m times have a mean of 54.2 s and a standard deviation of 0.9 s. Jess's times have a mean of 53.8 s and a standard deviation of 2.1 s. Who should the coach choose? Give reasons. [2 marks]

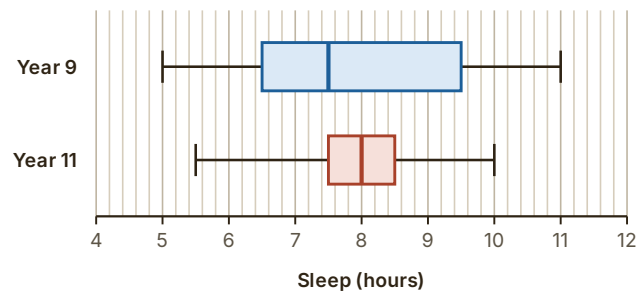
C8 Tom's standardised score in a reading test was 0. What does this tell you about his mark? [1 mark]

D Exam-style questions

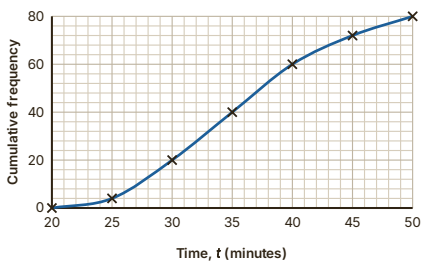
Give reasons in the context of the question: half of the marks are for what you write.

D1 The box plots show how many hours students in Year 9 and in Year 11 slept on one night. [3 marks]
Kim says, "The box for Year 9 is longer, so the Year 9 students slept more."

Is Kim right? Give a reason, using both box plots.



D2 The graph shows the cumulative frequency of the times that the 80 runners in Club A took to run 8 km.



(a) Find the median and the interquartile range of the times. [3 marks]

Answer Median min IQR min

(b) Estimate how many of the runners took more than 40 minutes. [1 mark]

Answer

- (c) The runners in Club B have a median time of 37 minutes and an interquartile range of 6 minutes. Compare the times of the two clubs. [2 marks]

Total for D2: 6 marks

- D3** Here are Sam's scores with three darts in 6 turns.

45, 60, 52, 38, 55, 50

- (a) Find Sam's mean score and the standard deviation of his scores. [3 marks]

Answer Mean s.d.

- (b) Lucy's scores have a mean of 48 and a standard deviation of 12.3. [2 marks]
Compare the scores of Sam and Lucy.

Total for D3: 5 marks

- D4** The annual incomes of the people in a town have a mean of £34 500, a median of £29 000 and a standard deviation of £11 000.

- (a) Work out the skewness of the incomes. [2 marks]

Answer

- (b) What does your answer to part (a) tell you about the incomes in the town? [1 mark]

- (c) An advert describes the "typical income" in the town. Which average should it use? Give a reason. [1 mark]

Total for D4: 4 marks

D5 Nina scored 72 in Maths, where the mean was 58 and the standard deviation was 8. She scored 66 in Science, where the mean was 54 and the standard deviation was 6.

- (a) Compared with the other students, did Nina do better in Maths or in Science? Show your working. [3 marks]

Answer

- (b) What Maths mark would have given Nina the same standardised score as her Science mark? [2 marks]

Answer

Total for D5: 5 marks

D6 The box plot shows the cost of each meal on a menu, in pounds. Describe the skew of the prices. Give a reason, and say what it means for the meals. [2 marks]



D7 A machine cuts metal rods. Their lengths have a mean of 250 mm and a standard deviation of 1.2 mm. A fault makes every rod 3 mm longer.

- (a) Write down the mean and the standard deviation of the lengths of the faulty rods, in mm. [2 marks]

Answer Mean mm s.d. mm

- (b) The new lengths are written in cm. Find their mean and standard deviation in cm. [1 mark]

Answer Mean cm s.d. cm

Total for D7: 3 marks

- D8** For the scores x of 20 players in a game, $\sum x = 1040$ and $\sum x^2 = 55\,520$. Work out the standardised score of a player who scored 64. [3 marks]

Answer

E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A group of 10 values has a mean of 12 and a standard deviation of 3. A second group of 15 values has a mean of 17 and a standard deviation of 4. The two groups are put together. Find the mean and the standard deviation of the 25 values together. [4 marks]

Answer Mean s.d.

- E2** Five whole numbers have a mean of 11, a median of 9, a mode of 7 and a range of 12. Work out the standard deviation of the five numbers. [4 marks]

Answer

- E3** In a test, the mean mark was 50 and Ravi's mark of 62 had a standardised score of 1.5. After a marking error was found, every mark was increased by 5 and then multiplied by 1.2. Work out Ravi's new standardised score. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can find the median, quartiles and interquartile range from a list or a graph.	☐	☐	☐
I can find outliers with the $1.5 \times \text{IQR}$ rule and compare two box plots in context.	☐	☐	☐
I can work out a standard deviation and say what changing the units does to it.	☐	☐	☐
I can work out and describe skew, and say what it means in context.	☐	☐	☐
I can work out standardised scores and use them to compare results.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1**
- Hint 1** Find $\sum x$ and $\sum x^2$ for each group.
- Hint 2** For the first group, $\sum x = 120$ and $\frac{\sum x^2}{10} - 12^2 = 3^2$, so $\sum x^2 = 1530$. The second group gives $\sum x = 255$ and $\sum x^2 = 4575$.
- Hint 3** Add the totals: $\sum x = 375$ and $\sum x^2 = 6105$ for $n = 25$. Then use the formula.
-
- E2**
- Hint 1** Write the numbers in order, a, b, c, d, e . Which of them can the mode be?
- Hint 2** The median is $c = 9$ and the 7 must appear twice, so $a = b = 7$. The range gives $e = 19$.
- Hint 3** The total is $5 \times 11 = 55$, so $d = 13$. The numbers are 7, 7, 9, 13, 19.
-
- E3**
- Hint 1** Find the standard deviation of the original marks first.
- Hint 2** The standard deviation was $12 \div 1.5 = 8$. The new mean is $(50 + 5) \times 1.2 = 66$ and the new standard deviation is $8 \times 1.2 = 9.6$.
- Hint 3** Ravi's new mark is $(62 + 5) \times 1.2 = 80.4$. Work out his new standardised score.