



Standard deviation and variance

About 50 minutes Calculator allowed 45 marks

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

A1 Needed today [2 marks] **A2** Last lesson [2 marks]

Work out $\sqrt[3]{\frac{125}{216}}$.

Answer

A box plot shows the masses, in grams, of a batch of tomatoes. Lowest value 28, lower quartile 32, median 45, upper quartile 50, highest value 80. Use the $1.5 \times \text{IQR}$ rule to decide whether the highest value is an outlier.

Answer

A3 From chapter 5 [2 marks] **A4** From chapter 7 [2 marks]

A cumulative frequency graph shows the ages of 160 members of a gym. Reading across from cumulative frequencies of 16, 32, 40, 80, 120, 128, 144 gives 15, 16, 28, 29, 30, 44, 58 years. Estimate the interquartile range.

Answer

Two groups of bakers recorded their masses, in grams, of loaves. Group A: range 24, interquartile range 17. Group B: range 48, interquartile range 12. One group contains a single value far from the rest. Which group's masses were more consistent? Give a reason.

B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Here are the numbers of goals scored in 6 hockey matches.

2, 5, 1, 3, 4, 3

Work out the variance and the standard deviation.

Hint Find the mean. Square each distance from the mean, add them, and divide by 6.

B1 Here are the times, in [3 marks]

minutes, that 8 students took to finish a puzzle.

12, 15, 9, 18, 14, 11, 16, 17

Work out the standard deviation of the times.

Answer

EXAMPLE 2

The table shows the numbers of pets owned by 20 people. Work out the mean and the standard deviation.

Pets	Frequency
0	5
1	8
2	4
3	3

Hint Work out $\sum fx$ and $\sum fx^2$. Standard deviation $= \sqrt{\frac{\sum fx^2}{n} - \bar{x}^2}$.

B2 The table shows the numbers [4 marks]

of eggs laid by 25 hens in one week. Work out the mean and the standard deviation.

Eggs	Frequency
2	3
3	7
4	10
5	5

Answer Mean s.d.

EXAMPLE 3

For 10 parcels, $n = 10$, $\sum x = 240$ and $\sum x^2 = 5960$, where x is the mass in kg. Work out the mean and the standard deviation.

Hint Mean $= \frac{\sum x}{n}$. Variance $= \frac{\sum x^2}{n} - \bar{x}^2$.

B3 For the heights x cm of 25 [3 marks]

sunflowers, $\sum x = 1150$ and $\sum x^2 = 53\,800$. Work out the mean and the standard deviation of the heights.

Answer Mean cm s.d. cm

EXAMPLE 4

Daily temperatures in a town have a mean of 15 °C and a standard deviation of 3 °C. To change to °F, each temperature is multiplied by 1.8 and then 32 is added. Find the mean and standard deviation in °F.

Hint Multiplying changes the mean and the spread. Adding moves every value by the same amount, so it changes only the mean.

B4 The prices of the meals in a café have a mean of £40 and a standard deviation of £6. Every price goes up by 10%, and then a service charge of £3 is added to each. Find the new mean and the new standard deviation. [3 marks]

Answer Mean £ s.d. £

EXAMPLE 5

Over a season, Amir scored a mean of 42 runs with a standard deviation of 8 runs. Ben scored a mean of 38 runs with a standard deviation of 21 runs. Compare their scores.

Hint Compare one average and one spread. Say what each means for the runs.

B5 Two machines fill bags of flour. Machine P: mean 502 g, standard deviation 1.8 g. Machine Q: mean 500 g, standard deviation 4.6 g. Compare the masses of the bags from the two machines. [2 marks]

c Practice

- C1** The masses of some apples have a mean of 64 g and a standard deviation of 7 g. An outlier is more than 3 standard deviations from the mean. Is either 45 g or 86 g an outlier? Show your working. [2 marks]

Answer Outlier:

- C2** Set A is 4, 7, 9, 12, 13. Set B is 24, 27, 29, 32, 33. Without working them out, explain why the two sets have the same standard deviation. [1 mark]

- C3** Eight numbers have a mean of 15 and a standard deviation of 4. Work out $\sum x^2$. [3 marks]

Answer

- C4** Nine values have a mean of 20 and a standard deviation of 6. A tenth value, 20, is added. Work out the new standard deviation. [3 marks]

Answer

D Exam-style questions

- D1** The workers in a shop are paid a mean of £11.40 an hour with a standard deviation of £1.30. Every worker gets a rise of £2 an hour. [3 marks]
- Sam says, "The standard deviation of the pay also goes up by £2."
- Is Sam right? Give the new mean and the new standard deviation, with a reason.

- D2** The table shows the lengths of 40 phone calls to a helpline.

Length t (minutes)	Frequency
$0 < t \leq 5$	12
$5 < t \leq 10$	18
$10 < t \leq 15$	7
$15 < t \leq 20$	3

- (a) Work out estimates of the mean and the standard deviation of the call lengths. [4 marks]

Answer Mean min s.d. min

- (b) At a second helpline, the calls have a mean of 9.1 minutes and a standard deviation of 2.8 minutes. Compare the lengths of the calls at the two helplines. [2 marks]

Total for D2: 6 marks

E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Ten numbers have a mean of 6 and a standard deviation of 2. Two of the numbers, 5 and 9, are removed. [4 marks]
- Work out the mean and the standard deviation of the eight numbers that are left.

Answer Mean s.d.

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** You cannot find the numbers. Find $\sum x$ and $\sum x^2$ for all ten instead.
- Hint 2** $\sum x = 60$. From $\frac{\sum x^2}{10} - 6^2 = 2^2$, $\sum x^2 = 400$.
- Hint 3** Take away $5 + 9$ and $5^2 + 9^2$: the eight left have $\sum x = 46$ and $\sum x^2 = 294$. Then use the formula with $n = 8$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out the variance and standard deviation from a list, a table or summary totals. ☐ ☐ ☐

I can say what adding or multiplying by a number does to the mean and standard deviation. ☐ ☐ ☐

I can compare consistency using the standard deviation, in context. ☐ ☐ ☐