



The normal distribution

About 55 minutes Calculator allowed 43 marks

Date _____

A Do Now

The first question is what today's lesson needs. The others bring back earlier topics.

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| <p>A1 Needed today [2 marks]</p> <p>In a fitness test, the mean was 51. Cara's number of sit-ups was 31, which gives a standardised score of -1.25. Work out the standard deviation.</p> <p style="text-align: right;">Answer</p> | <p>A2 Last lesson [2 marks]</p> <p>Of 10 seeds in a tray, each one germinates with probability 0.75, independently of the others. Find the probability that exactly 9 of them germinate. Give your answer to 3 decimal places.</p> <p style="text-align: right;">Answer</p> |
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| <p>A3 From chapter 11 [2 marks]</p> <p>Of 200 cars in a car park, 88 are petrol cars and the rest are electric cars. 64 of the petrol cars and 25 of the electric cars need new tyres. A car that needs new tyres is picked at random. Find the probability that it is one of the petrol cars.</p> <p style="text-align: right;">Answer</p> | <p>A4 From chapter 10 [2 marks]</p> <p>Noor wants to measure the agreement between two judges. The data are not measurements: two judges have put 11 items in rank order. Should Noor use Spearman's rank or the PMCC? Give a reason.</p> <p>_____</p> <p>_____</p> <p>_____</p> |
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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The heights of a group of adults are normally distributed with mean 170 cm and standard deviation 8 cm. Estimate the percentage of adults (a) between 162 cm and 178 cm, (b) between 154 cm and 186 cm, (c) taller than 186 cm.

Hint 68% are within 1 sd of the mean, 95% within 2 sd and 99.7% within 3 sd. Half of each lies on each side.

B1 The times taken to finish a puzzle are normally distributed with mean 30 seconds and standard deviation 4 seconds.

- (a) Estimate the percentage of times between 22 and 38 seconds. [1 mark]

Answer

- (b) Estimate the percentage of times less than 26 seconds. [2 marks]

Answer

Total for B1: 3 marks

EXAMPLE 2

The masses of bags of rice are normally distributed with mean 500 g and standard deviation 10 g. (a) Estimate the percentage of bags between 490 g and 520 g. (b) A shop has 2000 bags. Estimate how many are between 490 g and 520 g.

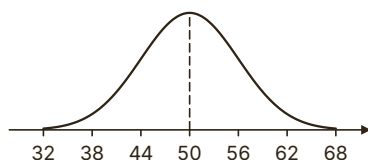
Hint The limits are not the same distance from the mean: work out each side on its own.

B2 The lengths of a type of leaf are normally distributed with mean 60 mm and standard deviation 5 mm. A student collects 400 leaves. Estimate how many of them are between 55 mm and 75 mm long. [3 marks]

Answer

EXAMPLE 3

The scores in a test are normally distributed with mean 50 and standard deviation 6. Sketch the distribution.



Hint The curve is a symmetrical bell centred on the mean. Mark the mean and 1, 2 and 3 sd either side, and end the tails at 3 sd.

B3 The lengths of some films are normally distributed with mean 120 minutes and standard deviation 15 minutes. You sketch this distribution. Write down the seven values you would mark on the scale, from 3 sd below the mean to 3 sd above. [2 marks]

EXAMPLE 4

A set of data is normally distributed. 95% of the values are between 32 and 48. Work out the mean and the standard deviation.

Hint The mean is halfway between the limits, and 95% covers 2 sd each side.

EXAMPLE 5

For the times that customers wait at a bank, the mean is 9 minutes, the median is 7 minutes and the mode is 5 minutes. Explain why a normal distribution would not be a good model for these times.

Hint In a normal distribution the mean, median and mode are equal.

B4 The masses of some apples [2 marks]

are normally distributed. 99.7% of the masses are between 133 g and 211 g. Work out the mean and the standard deviation.

B5 For the ages of the members [2 marks]

of a book club, the mean is 52 years, the median is 58 years and the mode is 63 years. Explain why a normal distribution would not be a good model for these ages.

c Practice

C1 The speeds of cars on a road [2 marks]
are normally distributed with mean 48 mph and standard deviation 3 mph. Estimate the percentage of cars travelling slower than 42 mph.

Answer

C2 The heights of 600 sunflowers [2 marks]
are normally distributed with mean 210 cm and standard deviation 12 cm. Estimate how many of the sunflowers are between 186 cm and 234 cm tall.

Answer

C3 The battery lives of some [2 marks]
phones are normally distributed with mean 18 hours and standard deviation 2 hours. Estimate the percentage of phones with a battery life between 18 and 24 hours.

Answer

C4 The masses of eggs are [2 marks]
normally distributed with mean 64 g. 97.5% of the eggs have a mass greater than 56 g. Work out the standard deviation.

Answer

D Exam-style questions

D1

The masses of some parcels are normally distributed with mean 40 kg and standard deviation 5 kg. Ellie says, "More than 85% of the parcels have a mass between 35 kg and 55 kg."

[5 marks]

Is Ellie correct? You must show your working.

D2 The lengths of phone calls to a help line are normally distributed with mean 18 minutes and standard deviation 3 minutes.

(a) On the axis, sketch this distribution. Label the scale.

[2 marks]

(b) Estimate the percentage of calls longer than 24 minutes.

[1 mark]

Answer

(c) The help line takes 400 calls in a week. Estimate how many of them last between 15 and 21 minutes.

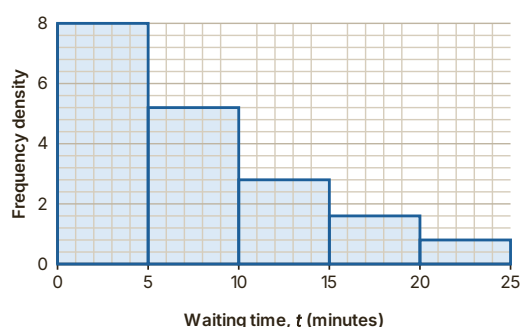
[2 marks]

Answer

Total for D2: 5 marks

D3 The histogram shows the waiting times of 92 patients at a clinic. Explain why a normal distribution would not be a suitable model for these times.

[2 marks]



E Extension

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The masses of jam in some jars are normally distributed with standard deviation 4 g. The label says 250 g. Only 0.15% of the jars contain less than 250 g. Work out the mean mass. [3 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** 0.15% is half of 100% – 99.7%.
Hint 2 So 250 g is 3 standard deviations below the mean.
Hint 3 Mean = $250 + 3 \times 4 = 262$ g.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use 68%, 95% and 99.7% to estimate percentages and numbers, including between limits that are not symmetrical. ○ ○ ○

I can sketch a normal curve with its scale and find the mean and standard deviation from the percentages. ○ ○ ○

I can explain why skewed data are not normally distributed. ○ ○ ○