

Time series, moving averages and seasonal variation



Hints online, answers free with an account:
mathswariz.uk/worksheets/stats-time-series-moving-averages-and-seasonal-variation/

About 80 minutes Calculator allowed 80 marks

Date _____

A Do Now

The first two are what this chapter needs. The rest bring back earlier chapters so nothing is forgotten.

A1 Needed today [2 marks]

The 18 plants in tray A have a mean height of 48 cm. The 10 plants in tray B have a mean height of 20 cm. Work out the mean height of all the plants in the two groups together.

Answer

A2 Needed today [2 marks]

Find the distance between the points $(-4, -2)$ and $(8, -11)$.

Answer

A3 Maths skill [2 marks]

Work out $\sqrt{1.96}$.

Answer

A4 Maths skill [2 marks]

Given that $27 \times 38 = 1026$, work out 2.7×0.38 .

Answer

A5

Maths skill

[2 marks]

A spinner can land on A, B, C or D. $P(A) = 0.2$, $P(B) = 0.2$ and $P(C) = 0.3$. It is spun 100 times. How many times would you expect it to land on D?

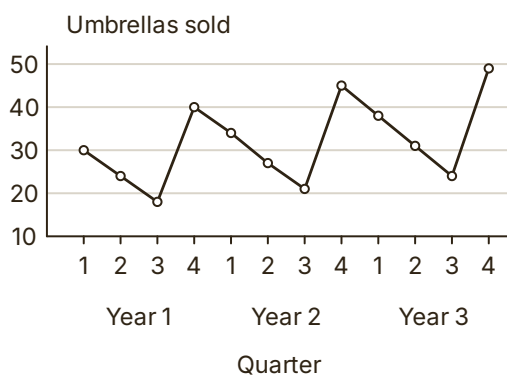
Answer

B Examples, then your turn

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

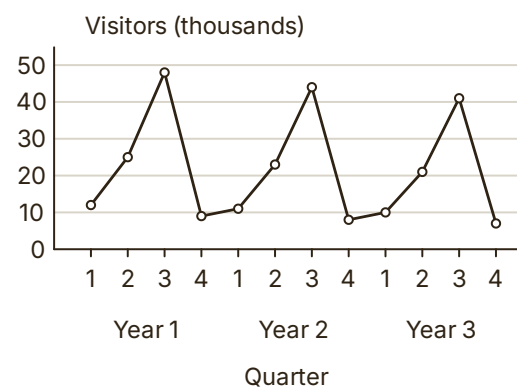
The graph shows a shop's umbrella sales in every quarter of a three-year period. Comment on how sales change over the years, and on the pattern within each year.



Hint The trend is the change from year to year; the seasonal pattern is what happens in the same quarter each year.

B1 The graph shows how many [2 marks]

people (in thousands) visited a theme park in every quarter of three years. Comment on the long-term movement and on the pattern inside each year.



EXAMPLE 2

The table gives a library's book loans per quarter. Find every 4-point moving average. At what point on the time axis does the second one go?

	Q1	Q2	Q3	Q4
Year 1	520	480	390	610
Year 2	540	500	410	630

Hint The second average uses Q2 of year 1 to Q1 of year 2.

- B2** The table gives bus passenger numbers (thousands) per quarter. Calculate the 4-point moving averages that start in Q1, Q2 and Q3 of year 1. At what point on the time axis is the second placed? [3 marks]

	Q1	Q2	Q3	Q4
Year 1	84	96	120	72
Year 2	88	100	124	76

Answer

EXAMPLE 3

A bakery is open Monday to Friday. The loaves it sold over two weeks were 40, 52, 47, 61, 75, 42, 55, 49, 63 and 79. Work out the first two 5-point moving averages, and say why 5 points are used.

Hint The pattern repeats every week of five days.

- B3** A hairdresser is open Tuesday to Saturday. Its customers on seven days in a row were 18, 24, 21, 30, 37, 20 and 26, starting on a Tuesday. Calculate the first two 5-point moving averages. Against which day is the first average plotted? [3 marks]

Answer

EXAMPLE 4

Over three years a cafe's Q2 sales came to 62, 70 and 75, against trend values of 55, 61 and 69. What is the mean seasonal variation for Q2?

Hint Find each seasonal variation as actual – trend, then take their mean.

- B4** In three successive years a florist took 118, 131 and 140 orders in Q4; the trend values for those quarters were 132, 141 and 155. Calculate the Q4 mean seasonal variation. [3 marks]

Answer

EXAMPLE 5

The first 4-point moving average, 210, sits midway between Q2 and Q3 of year 1. The ninth, 234, sits midway between Q2 and Q3 of year 3. Q1 has a mean seasonal variation of -18 . Assume the trend is a straight line. Predict the value for Q1 of year 4.

Hint Work out the change in the trend per quarter, then the trend for Q1 of year 4.

EXAMPLE 6

Explain why a prediction for next quarter is likely to be more reliable than a prediction for three years ahead.

Hint What has to stay the same for the prediction to work?

- B5** Two points on a straight trend line are the moving averages 96 (centred halfway through year 1, between its Q2 and Q3) and 84 (centred halfway through year 2). Q4 has a mean seasonal variation of 9. Assume the trend is a straight line. Predict the value for Q4 of year 3. [3 marks]

Answer

- B6** A shop used its trend line and the mean seasonal variation for Q4 to predict its Q4 sales. The actual sales were much lower. Give two possible reasons. [2 marks]

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C Practice

Each question asks for something different. Show your working.

- C1** A cinema sold 300, 420 and 360 tickets on the Friday, Saturday and Sunday of one weekend, and 291, 444 and 369 on the next. Work out the 3-point moving averages. [3 marks]

Answer

- C2** A 3-point moving average is 45. Two of its three values are 38 and 49. Work out the third value. [2 marks]

Answer

C3 A theatre records its ticket sales each month for five years. The pattern repeats every year. What order of moving average should it use? Give a reason. [2 marks]

C4 A 4-point moving average uses the values from Q3 of year 1 to Q2 of year 2. Between which two quarters is it plotted? [1 mark]

Answer

C5 In Q1 a cafe sold 3420 hot drinks, against a trend value of 2980. Find the seasonal variation for that quarter and explain what the number tells you. [2 marks]

C6 Over four years a farm shop's Q3 seasonal variations came to -22 , -18 , -25 and -19 . What is their mean? [2 marks]

Answer

C7 A straight trend line is drawn through two moving averages: 140, centred in the middle of year 1, and 164, centred in the middle of year 3. Estimate where the line is at Q3 of year 3. [2 marks]

Answer

C8 At Q4 of year 3 a trend line reads 250, and it drops 4 every quarter. Q3 has a mean seasonal variation of 22. Predict the value for Q3 of year 4. [3 marks]

Answer

D Exam-style questions

Answer every question in the space given. Show your working: most of the marks are for method.

D1

A shop sold 58 games consoles in Q2 of year 2. The mean of all twelve quarters of sales is 50, and the trend value for Q2 of year 2 is 54. Ella says, "The seasonal variation for Q2 of year 2 is $58 - 50 = 8$."

[2 marks]

Is Ella correct? Give a reason.

D2

A school's electricity use (MWh) over twelve quarters is in the table. Its nine 4-point moving averages, in order, are 56, 55, c , 53.5, 52, 51.25, d , 49.5 and 49.

	Q1	Q2	Q3	Q4
Year 1	64	50	41	69
Year 2	60	47	38	63
Year 3	57	43	35	61

(a) Calculate c and d .

[2 marks]

Answer

(b) Where on the time axis does the final average, 49, go?

[1 mark]

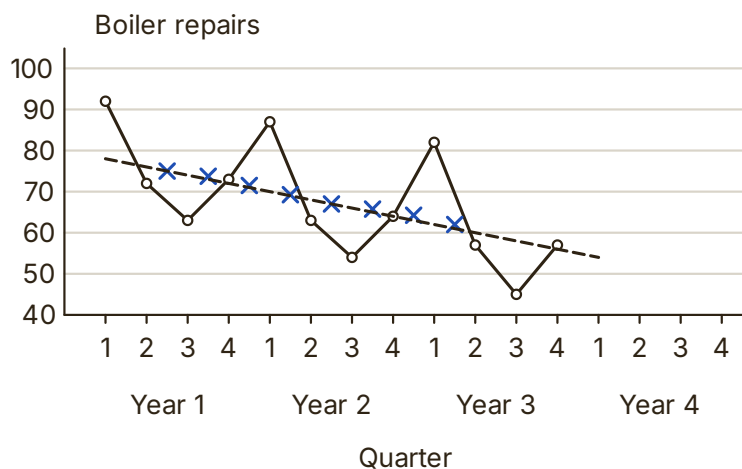
Answer

(c) What do the moving averages show about the trend? Use a figure in your answer.

[2 marks]

Total for D2: 5 marks

- D3** A firm's boiler repairs over twelve quarters are graphed. Its first eight 4-point moving averages are marked with crosses, and a dashed trend line runs on into year 4. The seasonal variations for Q1 in years 1 and 2 were 14 and 17.



- (a) Calculate the ninth 4-point moving average and say where on the time axis it belongs. [2 marks]

Answer

- (b) The firm made 82 repairs in Q1 of year 3. Read the trend line to find that quarter's seasonal variation. [1 mark]

Answer

- (c) Find the mean seasonal variation for Q1. [2 marks]

Answer

- (d) Predict the repairs in Q1 of year 4, reading the trend from the dashed line. [1 mark]

Answer

Total for D3: 6 marks

- D4** Tom has a shop's sales for each quarter of four years. He works out 3-point moving averages to show the trend. Explain why this is not a good choice, and say what he should use. [2 marks]

- D5** A hotel's bookings were 70, 92 and 101 in Q1, Q2 and Q3 of year 1. The 4-point moving average for Q1 to Q4 of year 1 is 85, and the next 4-point moving average is 87. Work out the bookings in Q4 of year 1 and in Q1 of year 2. [3 marks]

Answer

- D6** A firm's quarterly sales have a straight trend line through the moving averages 136 (middle of year 1) and 152 (middle of year 3). Q4 has a mean seasonal variation of -16 . [3 marks]
- (a) Predict the sales for Q4 of year 4.

Answer

- (b) The sales in Q4 of year 4 were 128. Comment on your prediction. [1 mark]

Total for D6: 4 marks

- D7** The mean seasonal variations for an ice cream van's sales are -18 for Q1, 6 for Q2, 25 for Q3 and -13 for Q4. [1 mark]
- (a) What does the mean seasonal variation for Q3 tell you?

- (b) Work out the total of the four mean seasonal variations. Explain why this total is what you would expect. [2 marks]

Total for D7: 3 marks

- D8** The 4-point moving averages for a gym's members are 412, 409, 405, 404, 398 and 395. [1 mark]
- (a) Describe the trend.

- (b) Work out the mean change in the moving averages per quarter. [2 marks]

Answer

Total for D8: 3 marks

E Extension

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** Consecutive 4-point moving averages of 68 and 72 are plotted half a quarter before and half a quarter after Q3 of year 1. That quarter had 85 sales. Using a centred moving average, find its seasonal variation. [3 marks]

Answer

- E2** A firm's prediction for Q1 next year is 145. The mean seasonal variation for Q1 is -23 , and the trend rises by 4 each quarter. Work out the trend value for Q3 this year. [3 marks]

Answer

- E3** Three 3-point moving averages in a row are 20, 22 and 25. The first two values in the list are 17 and 21. Work out the next three values. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can describe a time series and work out moving averages of the right order, plotted at the middle of their period.	☐	☐	☐
I can work out seasonal variations from the trend, and predict a value from the trend and the mean seasonal variation.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1	Hint 1	Neither average is plotted at Q3 itself. The mean of the two is.
	Hint 2	The centred moving average is $(68 + 72) \div 2 = 70$, plotted at Q3.
	Hint 3	The seasonal variation is $85 - 70$.

E2	Hint 1	Prediction = trend + mean seasonal variation. Find the trend for Q1 next year.
	Hint 2	The trend for Q1 next year is $145 + 23 = 168$.
	Hint 3	Q3 this year is 2 quarters before Q1 next year.

E3	Hint 1	Each moving average times 3 is the total of its three values.
	Hint 2	The totals are 60, 66 and 75. The first total gives the third value.
	Hint 3	Each new total adds one new value and drops the oldest.