



Congruence, transformations, constructions and loci

Grades 4–9 About 85 minutes

No calculator 90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 The point $(3, -2)$ is reflected in the x -axis. Write down the coordinates of its image. [1 mark]

Answer

A2 The point $(-4, 1)$ is translated by the vector $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$. Write down the coordinates of its image. [1 mark]

Answer

A3 Write down the equation of the straight line through $(2, 0)$ that is parallel to the y -axis. [1 mark]

Answer

A4 Name a quadrilateral that has rotational symmetry of order 2 but no lines of symmetry. [1 mark]

Answer

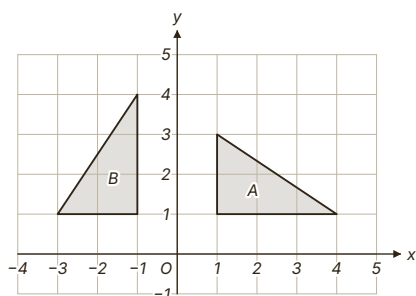
A5 A triangle has vertices $(1, 1)$, $(3, 1)$ and $(1, 2)$. Its image has vertices $(-1, 1)$, $(-3, 1)$ and $(-1, 2)$. Describe fully the single transformation. [2 marks]

Answer

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

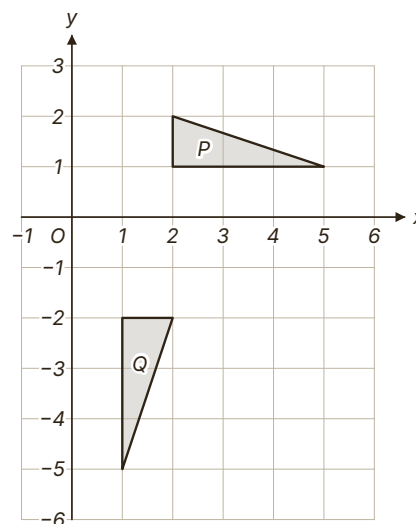
Triangle A has vertices $(1, 1)$, $(4, 1)$ and $(1, 3)$. Triangle B has vertices $(-1, 1)$, $(-1, 4)$ and $(-3, 1)$. Describe fully the single transformation that maps triangle A onto triangle B .



Hint A rotation needs three things: the angle, the direction and the centre. Use tracing paper to test a centre.

B1 Describe fully the single transformation that maps triangle P onto triangle Q .

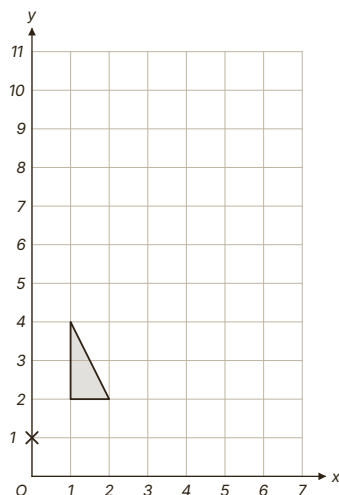
[3 marks]



Answer

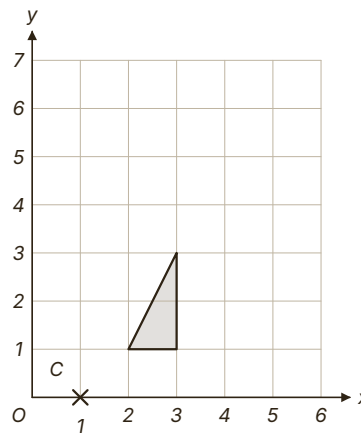
EXAMPLE 2

A triangle has vertices $(1, 2)$, $(2, 2)$ and $(1, 4)$. It is enlarged by scale factor 3, centre $(0, 1)$. Write down the coordinates of the vertices of the image.



Hint Find the vector from the centre to each vertex. Multiply it by the scale factor, and add it on to the centre.

B2 The shaded triangle is enlarged by scale factor 2, centre $C(1, 0)$. Write down the coordinates of the vertices of the image. [3 marks]



Answer

EXAMPLE 3

In triangle ABC , $AB = 7$ cm, $BC = 9$ cm and angle $ABC = 50^\circ$. In triangle DEF , $DE = 7$ cm, $EF = 9$ cm and angle $DEF = 50^\circ$. Are the triangles congruent? Give the reason.

Hint Check where the angle is. Is it between the two given sides?

B3 In triangle XYZ , $XY = 8$ cm, angle $X = 40^\circ$ and angle $Y = 75^\circ$. In triangle PQR , $PQ = 8$ cm, angle $P = 40^\circ$ and angle $Q = 75^\circ$. Are the triangles congruent? Give the reason. [2 marks]

Answer

EXAMPLE 4

Describe the locus of the points that are 3 cm from a fixed point P .

Hint Mark a few points 3 cm from P and look at the shape they make.

B4 A line segment AB is 6 cm long. [2 marks]

Describe the locus of the points that are 2 cm from AB .

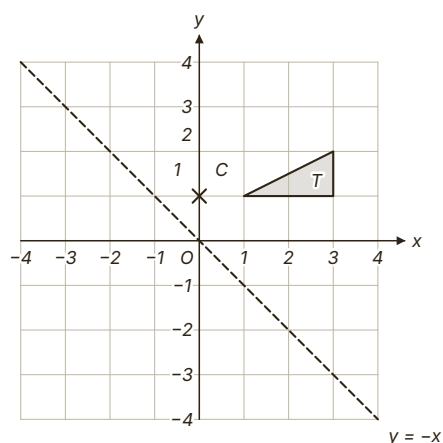
Answer

C Practice

Grades 5–7

Each question asks for something different. Show your working.

C1 Triangle T is shown on the grid.



- (a) Reflect triangle T in the line $y = -x$. Write down the coordinates of the vertices of the image. [2 marks]

Answer

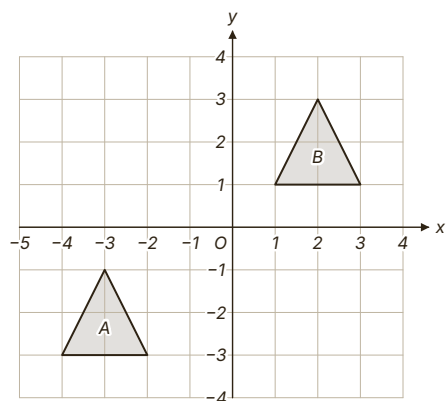
- (b) Rotate triangle T through 180° about the point $C(0, 1)$. Write down the coordinates of the vertices of the image. [2 marks]

Answer

Total for C1: 4 marks

C2 Describe fully the single transformation that maps triangle A onto triangle B .

[2 marks]

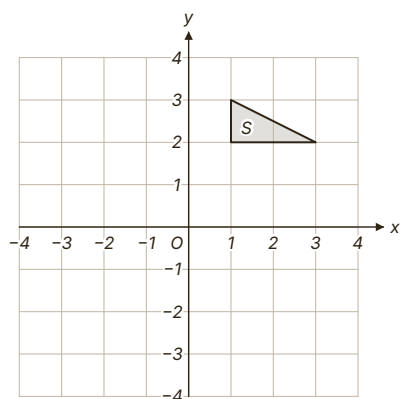


Answer

C3 Shape S is reflected in the x -axis. The image is then reflected in the y -axis.

[3 marks]

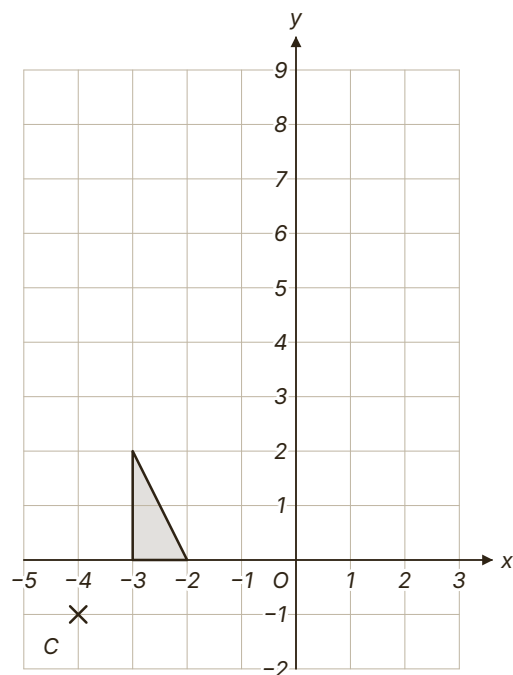
Describe fully the single transformation that maps shape S onto the final image.



Answer

- C4** The shaded triangle is enlarged by scale factor 3, centre $C(-4, -1)$. Write down the coordinates of the vertices of the image.

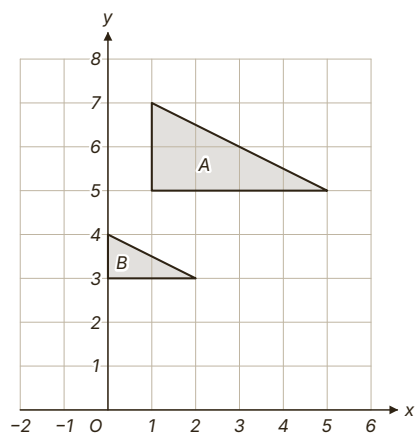
[3 marks]



Answer

- C5** Describe fully the single transformation that maps triangle A onto triangle B .

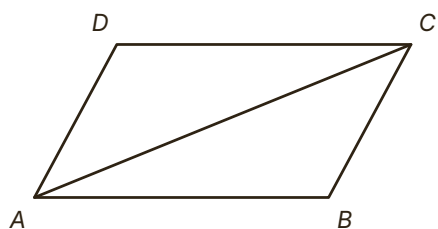
[3 marks]



Answer

- C6** $ABCD$ is a parallelogram. The diagonal AC is drawn.
Prove that triangle ABC is congruent to triangle CDA .

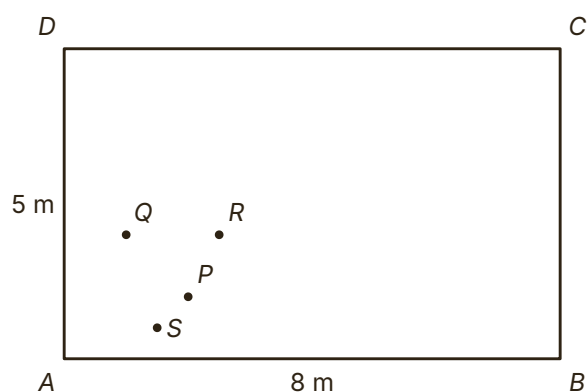
[3 marks]



- C7** The diagram shows a garden $ABCD$. A fountain is to be put in the garden so that it is less than 3 m from A and nearer to AB than to AD .

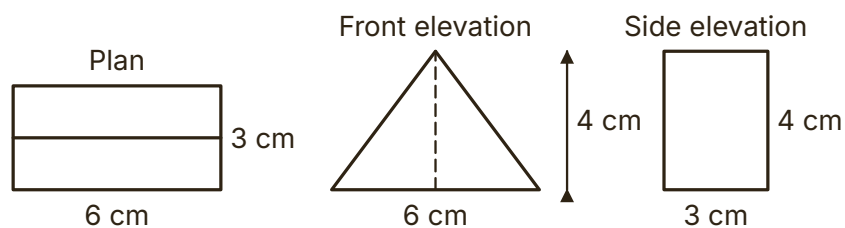
[3 marks]

Which of the points P , Q , R and S could be the fountain? You must show how you decide.



Answer

C8 The plan, front elevation and side elevation of a solid are shown.



(a) Write down the name of the solid.

[1 mark]

Answer

(b) Work out the volume of the solid.

[2 marks]

Answer

Total for C8: 3 marks

C9 Use a ruler and compasses to construct the perpendicular bisector of AB . You must show all your construction lines.

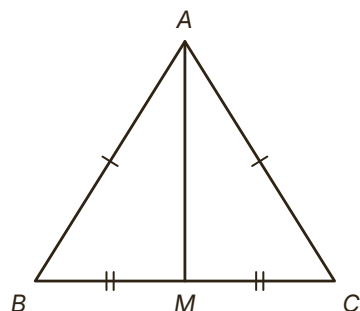
[2 marks]



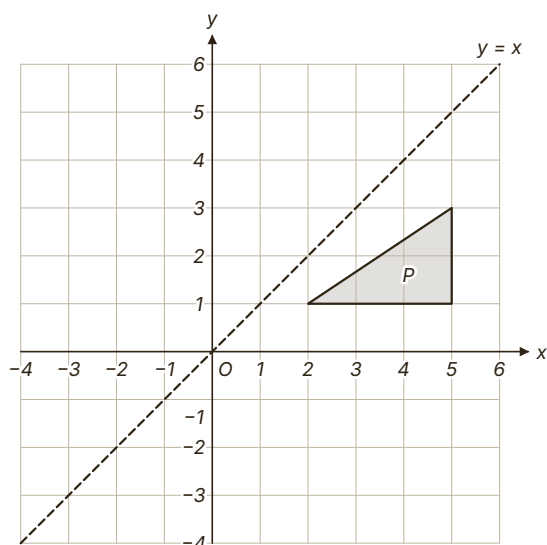
Answer every question in the space given. Show your working: most of the marks are for method.

- D1** ABC is an isosceles triangle with $AB = AC$. M is the midpoint of BC . [4 marks]

Prove that triangle ABM is congruent to triangle ACM , and hence show that AM is perpendicular to BC .



- D2** Triangle P is shown on the grid.



- (a) Reflect triangle P in the x -axis. Label the image Q . Write down the coordinates of the vertices of Q . [2 marks]

Answer

- (b) Reflect triangle Q in the line $y = x$. Label the image R . Write down the coordinates of the vertices of R . [2 marks]

Answer

- (c) Describe fully the single transformation that maps triangle P onto triangle R . [2 marks]

Answer

Total for D2: 6 marks

- D3** Tom says, "If two sides and an angle of one triangle are equal to two sides and an angle of another triangle, then the triangles must be congruent." [2 marks]

Tom is wrong. Explain why.

- D4** Two radio masts A and B are 8 km apart. A signal is picked up at points that are exactly 5 km from A and the same distance from A as from B . [3 marks]

There are two such points. Work out the distance between them.

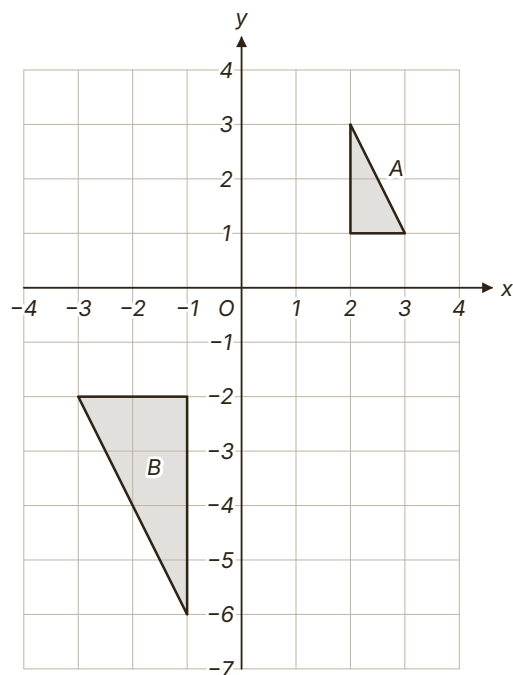


Not drawn accurately

Answer

D5 Describe fully the single transformation that maps triangle *A* onto triangle *B*.

[3 marks]



Answer

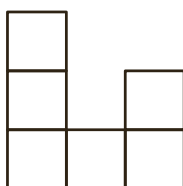
D6 A solid is made from centimetre cubes. It stands on a base 3 cm wide and 2 cm deep.

[3 marks]

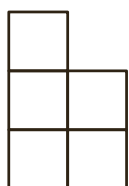
The diagram shows its front elevation and its side elevation.

Work out the largest number of cubes the solid could have.

Front elevation

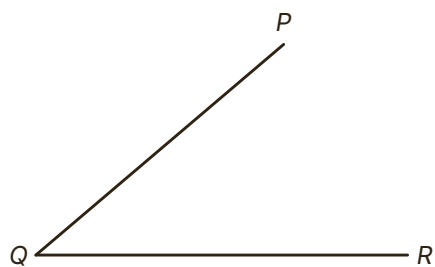


Side elevation



Answer

- D7** Use a ruler and compasses to construct the bisector of angle PQR . You must show all your construction lines. [2 marks]



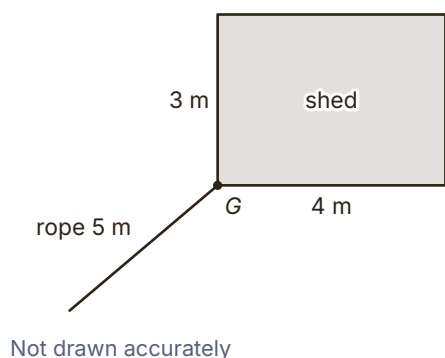
- D8** The point $P(5, 2)$ is rotated 90° clockwise about the point $(2, -1)$. The image is then translated by the vector $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$. [3 marks]

Work out the coordinates of the final image of P .

Answer (.....,)

- D9** A goat is tied to the corner G of a rectangular shed with a rope 5 m long. The shed is 4 m by 3 m. The goat cannot go into the shed. [4 marks]

Work out the area of ground the goat can reach. Give your answer in terms of π .

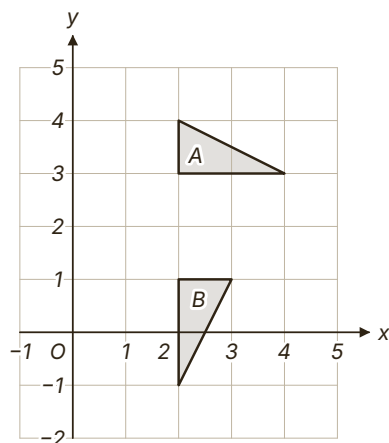


Not drawn accurately

Answer

D10 Describe fully the single transformation that maps triangle A onto triangle B .

[3 marks]



Answer

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** A shape is reflected in the line $x = 2$. The image is then reflected in the line $x = 6$. [3 marks]
Describe fully the single transformation that has the same effect.

Answer

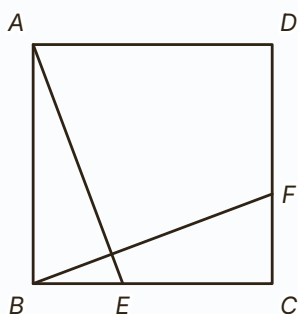
- E2** A shape is rotated 90° clockwise about $(0, 0)$. The image is then rotated 90° clockwise about $(4, 0)$. [4 marks]

Describe fully the single transformation that has the same effect.

Answer

- E3** $ABCD$ is a square. E is a point on BC and F is a point on CD , with $BE = CF$. [4 marks]

Prove that $AE = BF$ and that AE is perpendicular to BF .



- E4** An enlargement maps the point $(1, 2)$ to $(7, 8)$ and the point $(2, 0)$ to $(10, 2)$. [4 marks]

Find the scale factor and the coordinates of the centre of enlargement.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can prove that two triangles are congruent using SSS, SAS, ASA or RHS.	☐	☐	☐
I can carry out and describe rotations, reflections, translations and enlargements, and combine two of them.	☐	☐	☐
I can construct bisectors, find loci and regions, and use plans and elevations.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1**
- Hint 1** Try it with one point, such as $(0, 0)$, and then with another point.
- Hint 2** $(0, 0)$ goes to $(4, 0)$, then to $(8, 0)$.
- Hint 3** Every point moves 8 to the right. The general point (x, y) goes to $(4 - x, y)$ and then to $(12 - (4 - x), y) = (x + 8, y)$.
-
- E2**
- Hint 1** Follow a general point (x, y) through both rotations.
- Hint 2** About $(0, 0)$, (x, y) goes to $(y, -x)$. About $(4, 0)$, (a, b) goes to $(4 + b, 4 - a)$.
- Hint 3** So (x, y) ends at $(4 - x, 4 - y)$. A point and its image always have midpoint $(2, 2)$, so this is a half turn about $(2, 2)$.
-
- E3**
- Hint 1** Look for two congruent triangles, one with side AE and one with side BF .
- Hint 2** Triangles ABE and BCF : $AB = BC$, $BE = CF$ and the angles at B and C are both 90° .
- Hint 3** So the triangles are congruent (SAS) and $AE = BF$. Angle $BAE = \text{angle } CBF$. In triangle ABE , angle $BAE + \text{angle } AEB = 90^\circ$, so angle $CBF + \text{angle } AEB = 90^\circ$, and the angle where the lines cross is 90° .
-
- E4**
- Hint 1** Compare the vector between the two points with the vector between their images.
- Hint 2** $(1, 2) \rightarrow (2, 0)$ is $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ and $(7, 8) \rightarrow (10, 2)$ is $\begin{pmatrix} 3 \\ -6 \end{pmatrix}$, so the scale factor is 3.
- Hint 3** If the centre is (a, b) , then $7 = a + 3(1 - a)$ and $8 = b + 3(2 - b)$.