

Trigonometry in 2D and 3D, trigonometric graphs, and the sine and cosine rules



Hints online, answers free with an account:
mathswariz.uk/worksheets/triangles/

Grades 4–9 About 90 minutes Calculator allowed 90 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

- A1** A right-angled triangle has hypotenuse 10 cm. One of its angles is 35° . Find the length of the side opposite the 35° angle. Give your answer to 3 significant figures. [2 marks]

Answer

- A2** The two shorter sides of a right-angled triangle are 7 cm and 24 cm. Find the length of the hypotenuse. [1 mark]

Answer

- A3** In a right-angled triangle, the side opposite angle θ is 5 cm and the side next to it is 8 cm. Find θ to 1 decimal place. [2 marks]

Answer

- A4** Write down the exact value of $\sin 30^\circ$. [1 mark]

Answer

- A5** A triangle has base 9 cm and perpendicular height 6 cm. Find its area. [1 mark]

Answer

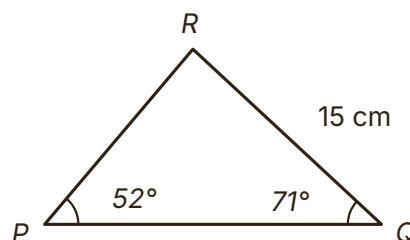
Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

In triangle ABC , angle $A = 48^\circ$, angle $B = 67^\circ$ and $BC = 12$ cm. Find the length of AC . Give your answer to 3 significant figures.

Hint Each side is opposite an angle. Use $\frac{AC}{\sin B} = \frac{BC}{\sin A}$.

B1 Find the length of PR . Give your answer to 3 significant figures. [3 marks]



Not drawn accurately

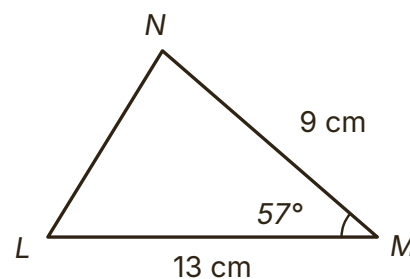
Answer

EXAMPLE 2

In triangle ABC , $AB = 11$ cm, $AC = 8$ cm and angle $BAC = 40^\circ$. Find the length of BC . Give your answer to 3 significant figures.

Hint You know two sides and the angle between them. Use $a^2 = b^2 + c^2 - 2bc \cos A$.

B2 Find the length of LN . Give your answer to 3 significant figures. [3 marks]



Not drawn accurately

Answer

EXAMPLE 3

A triangle has sides 7 cm, 9 cm and 12 cm. Find the size of its largest angle. Give your answer to 1 decimal place.

Hint The largest angle is opposite the longest side.

Rearrange the cosine rule: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

- B3** A triangle has sides 6 cm, 10 cm and 13 cm. Find the size of its largest angle. Give your answer to 1 decimal place. [3 marks]

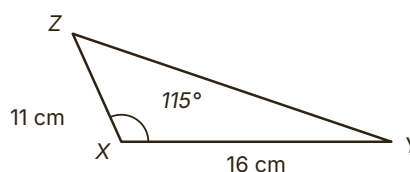
Answer

EXAMPLE 4

Find the area of a triangle with sides 14 cm and 9 cm and an angle of 38° between them. Give your answer to 3 significant figures.

Hint Area = $\frac{1}{2}ab \sin C$, where C is the angle between sides a and b .

- B4** Find the area of the triangle. Give your answer to 3 significant figures. [2 marks]



Not drawn accurately

Answer

Each question asks for something different. Show your working.

C1 Answer these without drawing a graph. Use the symmetry of the sine and cosine graphs.

- (a) $\sin 34^\circ = 0.559$ to 3 decimal places. Write down the other angle between 0° and 180° [1 mark] whose sine is 0.559.

Answer

- (b) Write down the angle x between 180° and 360° for which $\cos x = \cos 70^\circ$. [1 mark]

Answer

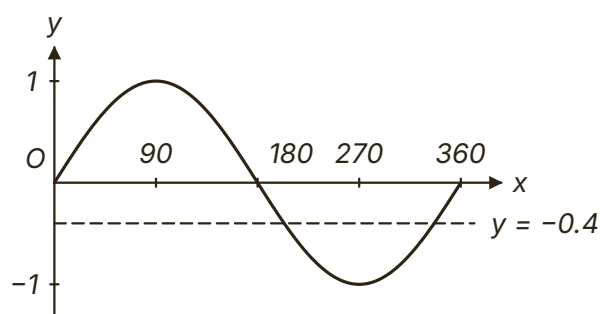
- (c) Write down the coordinates of the lowest point on the graph of $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$. [1 mark]

Answer (.....,)

Total for C1: 3 marks

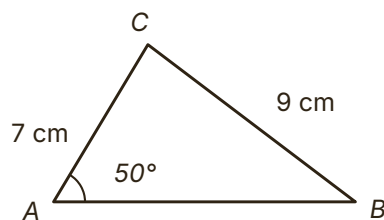
C2 The graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ is drawn with the line $y = -0.4$. [3 marks]

Solve $\sin x = -0.4$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place.



Answer

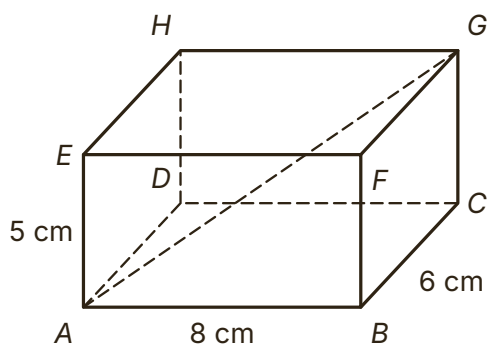
- C3** In triangle ABC , $BC = 9$ cm, $AC = 7$ cm and angle $BAC = 50^\circ$. Find the size of angle ABC . Give your answer to 1 decimal place. [3 marks]



Not drawn accurately

Answer

- C4** The diagram shows a cuboid $ABCDEFGH$. $AB = 8$ cm, $BC = 6$ cm and $AE = 5$ cm.



Not drawn accurately

- (a) Find the length of the diagonal AG . Give your answer to 3 significant figures. [2 marks]

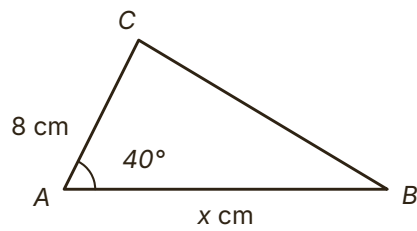
Answer

- (b) Find the angle between AG and the base $ABCD$. Give your answer to 1 decimal place. [2 marks]

Answer

Total for C4: 4 marks

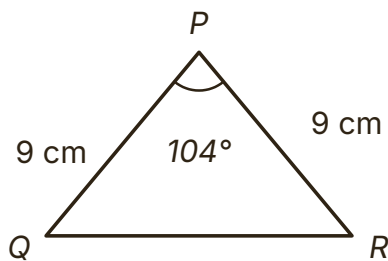
- C5** The area of the triangle is 30 cm^2 . Find the value of x . Give your answer to 3 significant figures. [3 marks]



Not drawn accurately

Answer

- C6** Triangle PQR is isosceles with $PQ = PR = 9 \text{ cm}$ and angle $QPR = 104^\circ$. Find the length of QR . Give your answer to 3 significant figures. [3 marks]

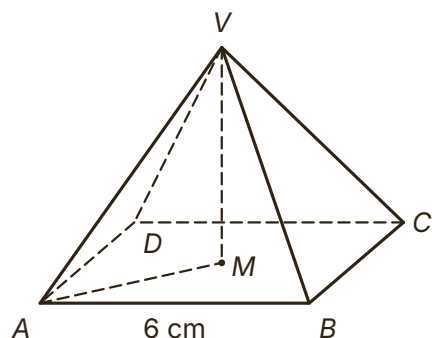


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Answer

- C7** $VABCD$ is a pyramid with a square base of side 6 cm. The vertex V is directly above M , the centre of the base, and $VM = 8$ cm. [3 marks]

Find the angle between the edge VA and the base. Give your answer to 1 decimal place.



Not drawn accurately

Answer

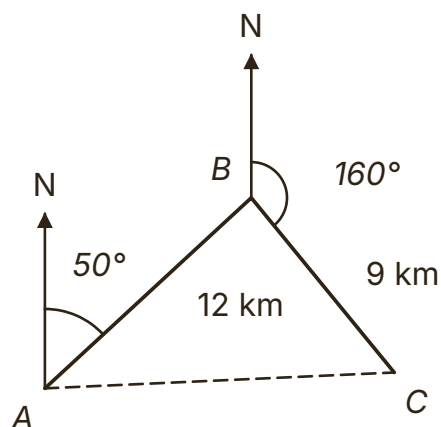
- C8** Two sides of a triangle are 7 cm and 10 cm. The angle between them is 120° . Find the perimeter of the triangle. Give your answer to 3 significant figures. [3 marks]

Answer

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** A boat sails 12 km from A to B on a bearing of 050° . It then sails 9 km from B to C on a bearing of 160° . [4 marks]

Work out the distance AC . Give your answer to 3 significant figures.



Not drawn accurately

Answer

- D2** Ravi says, "The equation $\sin x = 0.5$ has only one solution between 0° and 360° . It is $x = 30^\circ$." [2 marks]

Ravi is wrong. Explain why, and give all the solutions between 0° and 360° .

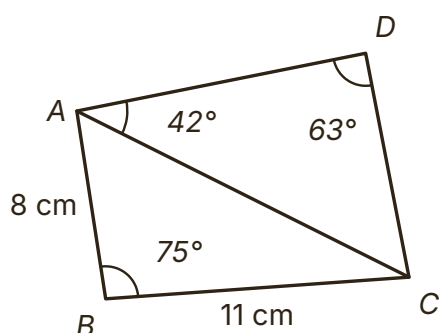
Answer

D3 In triangle ABC , $AB = 11$ cm, $BC = 8$ cm and angle $BAC = 42^\circ$. [3 marks]

Two different triangles fit this information. Find the two possible sizes of angle ACB .
Give your answers to 1 decimal place.

Answer

D4 The diagram shows a quadrilateral $ABCD$. $AB = 8$ cm, $BC = 11$ cm, angle $ABC = 75^\circ$, angle $CAD = 42^\circ$ and angle $ADC = 63^\circ$.



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(a) Work out the length of AC . Give your answer to 3 significant figures. [2 marks]

Answer

(b) Work out the length of CD . Give your answer to 3 significant figures. [2 marks]

Answer

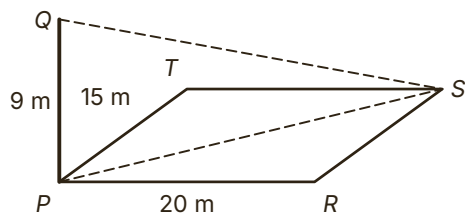
(c) Work out the area of $ABCD$. Give your answer to 3 significant figures. [2 marks]

Answer

Total for D4: 6 marks

- D5** $PRST$ is a horizontal rectangular field with $PR = 20$ m and $PT = 15$ m. A vertical flagpole PQ of height 9 m stands at the corner P . [3 marks]

Find the angle of elevation of Q from the opposite corner S . Give your answer to 1 decimal place.



Not drawn accurately

Answer

- D6** A triangle has sides 8 cm and 9 cm. Its area is 25 cm^2 . [3 marks]

Find the two possible sizes of the angle between the 8 cm and 9 cm sides. Give your answers to 1 decimal place.

Answer

- D7** A triangle has sides 7 cm, 8 cm and 13 cm. [3 marks]

Show that its largest angle is 120° .

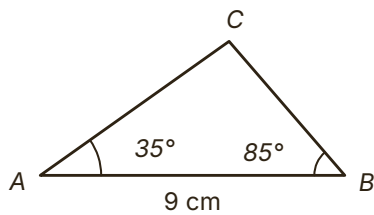
- D8** Solve $3 \cos x + 1 = 0$ for $0^\circ \leq x \leq 360^\circ$. Give your answers to 1 decimal place. [3 marks]

Answer

D9 In triangle ABC , angle $A = 35^\circ$, angle $B = 85^\circ$ and $AB = 9$ cm.

[4 marks]

Work out the area of the triangle. Give your answer to 3 significant figures.



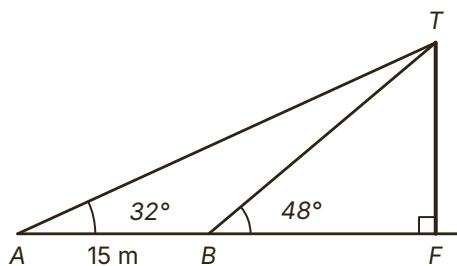
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Answer

D10 A and B are points on level ground in a straight line with the foot F of a vertical tower FT . $AB = 15$ m. The angle of elevation of T from A is 32° and from B is 48° .

[4 marks]

Work out the height of the tower. Give your answer to 3 significant figures.



Not drawn accurately

Answer

E Extension

Grade 9 and beyond

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

E1 In triangle ABC , $AB = 2\sqrt{3}$ cm, $AC = 5$ cm and angle $BAC = 30^\circ$. [3 marks]

Find the exact length of BC .

Answer

E2 Find every value of x between 0° and 360° for which $\sin x = \cos x$. [3 marks]

Answer

E3 In triangle ABC , $AB = 12$ cm, angle $BAC = 30^\circ$ and $BC = k$ cm. [3 marks]

Find the range of values of k for which two different triangles can be drawn.

Answer

E4 Find the angle between a diagonal of a cube and one of its faces. Give your answer to 1 decimal place. [3 marks]

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can use trigonometry and Pythagoras in several steps, in 2D and 3D, and find the angle between a line and a plane.	☐	☐	☒
I can use the sine and cosine graphs to find the second solution a calculator does not give.	☐	☐	☒
I can use the sine rule and the cosine rule to find a side or an angle, and choose between them.	☐	☐	☒
I can find the area of any triangle with $\frac{1}{2}ab \sin C$.	☐	☐	☒

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

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- | | | |
|-----------|---------------|--|
| E1 | Hint 1 | You know two sides and the angle between them, and $\cos 30^\circ$ has an exact value. |
| | Hint 2 | $BC^2 = (2\sqrt{3})^2 + 5^2 - 2 \times 2\sqrt{3} \times 5 \times \frac{\sqrt{3}}{2}$ |
| | Hint 3 | $BC^2 = 12 + 25 - 30 = 7$, so $BC = \sqrt{7}$ cm. |
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- | | | |
|-----------|---------------|---|
| E2 | Hint 1 | Sketch both graphs on the same axes and look for where they cross. |
| | Hint 2 | They cross where $\tan x = 1$, which is at 45° . |
| | Hint 3 | The graphs repeat the crossing 180° later, where both are negative. So $x = 45^\circ$ or $x = 225^\circ$. |
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- | | | |
|-----------|---------------|--|
| E3 | Hint 1 | Draw AB and the line from A at 30° . Then swing an arc of radius k from B and see how many times it meets the line. |
| | Hint 2 | The shortest distance from B to the line is $12 \sin 30^\circ = 6$. |
| | Hint 3 | The arc meets the line twice on the correct side of A when it is longer than 6 but shorter than AB . So $6 < k < 12$. |
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- | | | |
|-----------|---------------|---|
| E4 | Hint 1 | Call the side of the cube 1. Find the length of the diagonal of a face. |
| | Hint 2 | The face diagonal is $\sqrt{2}$, and the cube's diagonal rises 1 above it. |
| | Hint 3 | $\tan \theta = \frac{1}{\sqrt{2}}$, so $\theta = 35.3^\circ$. |
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