



Hints online, answers free with an account:
mathswariz.uk/worksheets/variation/

Direct and inverse proportion: formulae, graphs and changes

Grades 4–9 About 75 minutes Calculator allowed 88 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 $y = 4x$. Find the value of y when $x = 7.5$.

[1 mark]

Answer _____

A2 Find the value of k when $45 = k \times 3^2$.

[1 mark]

Answer _____

A3 Work out $6 \div 0.25$.

[1 mark]

Answer _____

A4 Make x the subject of $y = kx^2$, where x and k are positive.

[2 marks]

Answer _____

A5 Work out $\sqrt[3]{343}$.

[1 mark]

Answer _____

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

y is directly proportional to x . When $x = 6$, $y = 42$. Find y when $x = 11$.

Hint Write $y = kx$. Use the pair of values you are given to find k .

B1 P is directly proportional to Q . [3 marks]

When $Q = 12$, $P = 54$.

Find a formula for P in terms of Q . Then find P when $Q = 20$.

Answer $P = \dots\dots$ $P = \dots\dots$

EXAMPLE 2

y is directly proportional to x^2 . When $x = 3$, $y = 18$. Find the positive value of x when $y = 50$.

Hint Write $y = kx^2$ and find k . Then put $y = 50$ in and solve for x .

B2 A is directly proportional to the [3 marks]

square of r . When $r = 4$, $A = 48$. Find the positive value of r when $A = 147$.

Answer

EXAMPLE 3

y is inversely proportional to x . When $x = 5$, $y = 12$. Find y when $x = 4$.

Hint Write $y = \frac{k}{x}$. Then $k = xy$.

B3 The time, T hours, it takes to [3 marks]

paint a fence is inversely proportional to the number of painters, n . 6 painters take 8 hours.

How long do 4 painters take?

Answer

EXAMPLE 4

y is inversely proportional to x^2 . When $x = 4$, $y = 5$. Find y when $x = 10$.

Hint Write $y = \frac{k}{x^2}$, so $k = x^2y$.

B4 F is inversely proportional to the square of d . When $d = 5$, $F = 36$. Find the positive value of d when $F = 4$. [3 marks]

Answer

C Practice

Grades 5–7

Each question asks for something different. Show your working.

C1 In the table, y is directly proportional to x . Find the value of a .

[2 marks]

x	4	10	14
y	10	25	a

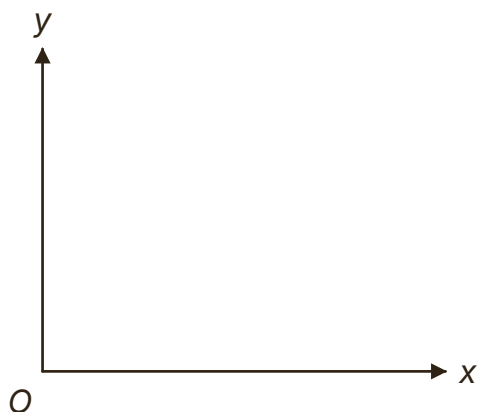
Answer

C2 y is directly proportional to the square root of x . When $x = 9$, $y = 15$. Find y when $x = 64$.

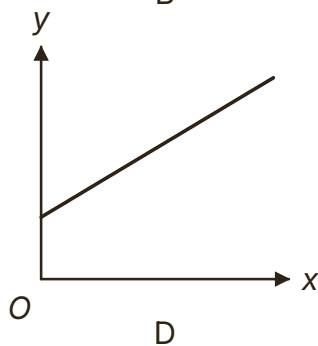
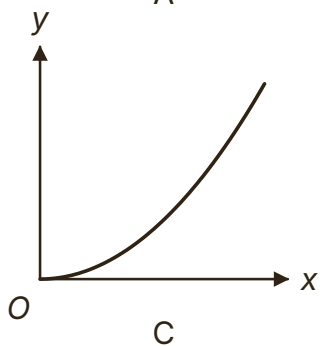
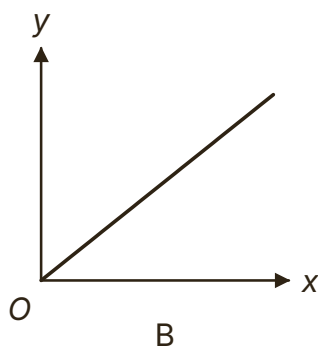
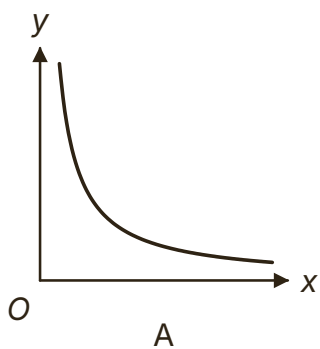
[3 marks]

Answer

- C3** y is inversely proportional to x . On the axes, sketch the graph of y against x for $x > 0$. [2 marks]



- C4** Here are four sketch graphs.



Not drawn accurately

- (a)** Which graph shows y directly proportional to x ? [1 mark]

Answer

- (b)** Which graph shows y directly proportional to x^2 ? [1 mark]

Answer

(c) Which graph shows y inversely proportional to x ?

[1 mark]

Answer

Total for C4: 3 marks

C5 Each part is about what happens to y when x changes.

(a) y is directly proportional to x^3 . x is doubled. What number is y multiplied by?

[1 mark]

Answer

(b) y is inversely proportional to x^2 . x is multiplied by 4. What number is y multiplied by?

[1 mark]

Answer

Total for C5: 2 marks

C6 The frequency, f hertz, of a vibrating string is inversely proportional to its length, L metres. When $L = 0.6$, $f = 330$.

[3 marks]

Find f when $L = 0.45$.

Answer

C7 The table shows some values of x and y .

[2 marks]

Is y inversely proportional to x , or to x^2 ? Give a reason for your answer.

x	1	2	4
y	64	16	4

C8 V is directly proportional to the cube of h . When $h = 4$, $V = 160$.

(a) Find a formula for V in terms of h .

[2 marks]

Answer

(b) Find V when $h = 6$.

[1 mark]

Answer

(c) Find h when $V = 20$.

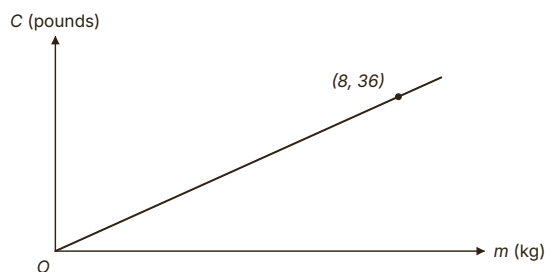
[2 marks]

Answer

Total for C8: 5 marks

C9 The cost, C pounds, of sand is directly proportional to its mass, m kg. The graph shows this. [2 marks]

Work out the cost of 15 kg of sand.



Answer

D Exam-style questions

Grades 6–9

Answer every question in the space given. Show your working: most of the marks are for method.

D1 The cost of a round table top is directly proportional to the square of its diameter. A table top of diameter 75 cm costs £90. [3 marks]

Find the cost of a table top of diameter 1.2 m.

Answer

D2 Mia says, " y is inversely proportional to x^2 , so when x is doubled, y is halved." [2 marks]

Mia is wrong. Explain why, and say what happens to y when x is doubled.

Answer

D3 y is directly proportional to x^3 . [3 marks]

Show that when x is increased by 20%, y is increased by 72.8%.

D4 The pressure, P , of a gas is inversely proportional to its volume, V . When $V = 1.5$, $P = 240$.

(a) Find P when $V = 2.4$. [2 marks]

Answer

(b) Find V when $P = 320$. [2 marks]

Answer

Total for D4: 4 marks

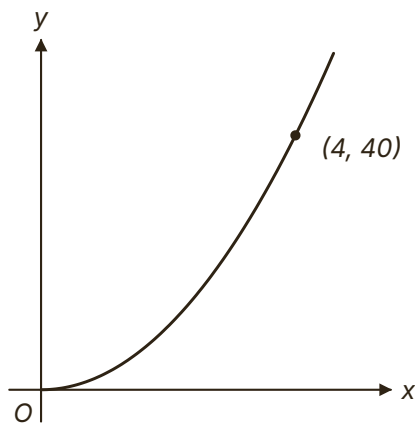
D5 a is directly proportional to b^2 . b is inversely proportional to c . [4 marks]

When $c = 2$, $b = 6$ and $a = 72$.

Find the value of a when $c = 3$.

Answer

- D6** The graph shows part of the curve $y = kx^2$. The curve passes through the point $(4, 40)$. [3 marks]
Find y when $x = 10$.



Not drawn accurately

Answer

- D7** A ball rolls down a slope from rest. The distance, d metres, it has rolled is directly proportional to the square of the time, t seconds, since it started. In the first 2 seconds it rolls 1.8 metres.
- (a) Find a formula for d in terms of t . [2 marks]

Answer

- (b) How far does the ball roll during the third second? [2 marks]

Answer

- (c) The slope is 20 metres long. How long does the ball take to reach the bottom? Give your answer to 3 significant figures. [2 marks]

Answer

Total for D7: 6 marks

- D8** The time, T seconds, for one swing of a pendulum is directly proportional to the square root of its length, L cm. A pendulum of length 64 cm takes 1.6 seconds. [3 marks]
- Find the length of a pendulum that takes 2.2 seconds.

Answer

- D9** The table shows some values of x and y . [3 marks]
- Find a formula for y in terms of x . You must show your working.

x	2	3	5
y	45	20	7.2

Answer

- D10** y is directly proportional to x^n . When $x = 2$, $y = 24$. When $x = 4$, $y = 192$. [3 marks]
- Find the value of n .

Answer

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** p is directly proportional to the square of q . q is inversely proportional to the cube of r . [3 marks]

r is doubled. What happens to p ?

Answer

- E2** The surface area of a sphere is proportional to the square of its radius. Its volume is proportional to the cube of its radius. [3 marks]

A balloon is a sphere. Its volume increases by 33.1%. By what percentage does its surface area increase?

Answer

- E3** The graph of $y = \frac{k}{x}$ passes through the points $(a, 18)$ and $(3a, a)$, where $a > 0$. [3 marks]
Find the value of a and the value of k .

Answer $a = \dots\dots$ $k = \dots\dots$

E4 a is inversely proportional to $\sqrt{b}$. b is increased by 44%. [3 marks]

Find the percentage decrease in a . Give your answer to 3 significant figures.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can write a direct proportion as a formula, including $y = kx^2$, $y = kx^3$ and $y = k\sqrt{x}$, find k and use it.	☐	☐	☒
I can do the same for inverse proportion, $y = \frac{k}{x}$ and $y = \frac{k}{x^2}$, and recognise each graph.	☐	☐	☒
I can work out what a change in one quantity does to the other.	☐	☐	☒

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

E1 **Hint 1** Write each relationship with its own constant, then put one into the other.

Hint 2 $q = \frac{k}{r^3}$, so $p = mq^2 = \frac{mk^2}{r^6}$.

Hint 3 p is inversely proportional to r^6 , and $2^6 = 64$, so p is divided by 64.

E2 **Hint 1** Find the number the radius is multiplied by first.

Hint 2 The volume is multiplied by 1.331, so the radius is multiplied by $\sqrt[3]{1.331}$.

Hint 3 $\sqrt[3]{1.331} = 1.1$, so the area is multiplied by $1.1^2 = 1.21$. That is an increase of 21%.

E3 **Hint 1** Each point gives an expression for k .

Hint 2 $k = 18a$ and $k = 3a \times a = 3a^2$.

Hint 3 $3a^2 = 18a$ and $a > 0$, so $a = 6$ and $k = 108$.

E4 **Hint 1** Find the number $\sqrt{b}$ is multiplied by.

Hint 2 $\sqrt{1.44} = 1.2$, so a is multiplied by $\frac{1}{1.2}$.

Hint 3 $\frac{1}{1.2} = 0.8333\dots$, which is a decrease of 16.7%.