



Column vectors, vector arithmetic and vector proofs in geometry

Grades 4–9 About 85 minutes No calculator 92 marks

Name _____

Class _____

Date _____

A Before you start

Grades 4–5

You will need these in every question below.

A1 $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$. Write $4\mathbf{a}$ as a column vector.

[1 mark]

Answer _____

A2 Work out the length of the vector $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$.

[1 mark]

Answer _____

A3 $\mathbf{a} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Write $2\mathbf{a} + 3\mathbf{b}$ as a column vector.

[2 marks]

Answer _____

A4 P is the point $(1, 2)$ and Q is the point $(7, -1)$. Write $\overrightarrow{PQ}$ as a column vector.

[1 mark]

Answer _____

- A5** The point $A(2, 5)$ is translated by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. Write down the coordinates of its image. [1 mark]

Answer

B Examples, then your turn

Grades 5–6

Your teacher works through each example with you. Then try the one beside it on your own.

EXAMPLE 1

$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. Write $2\mathbf{a} - 3\mathbf{b}$ as a column vector.

Hint Multiply each vector first, then subtract the top numbers and the bottom numbers separately.

- B1** $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$. Write $3\mathbf{p} + 2\mathbf{q}$ as a column vector. [2 marks]

Answer

EXAMPLE 2

$\mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$. $2\mathbf{c} + \mathbf{a} = \mathbf{b}$. Find $\mathbf{c}$ as a column vector.

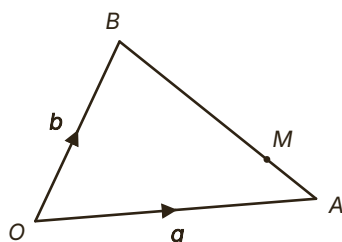
Hint Rearrange as you would an equation: $2\mathbf{c} = \mathbf{b} - \mathbf{a}$.

- B2** $\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$. $3\mathbf{c} - \mathbf{a} = \mathbf{b}$. Find $\mathbf{c}$ as a column vector. [3 marks]

Answer

EXAMPLE 3

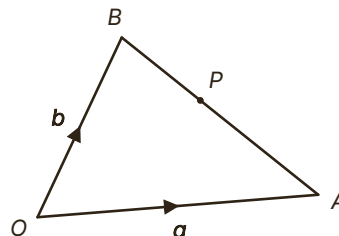
OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the point on AB with $AM : MB = 1 : 3$. Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.



Not drawn accurately

Hint Go from O to A , then a quarter of the way from A to B . $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

B3 OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is the point on AB with $AP : PB = 3 : 2$. Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Simplify your answer. [3 marks]



Not drawn accurately

Answer

EXAMPLE 4

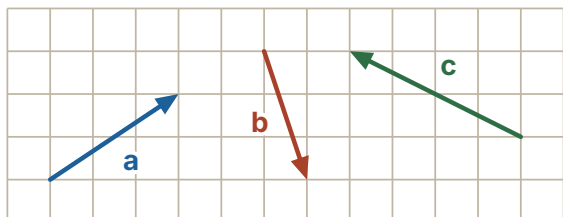
$\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} -9 \\ 6 \end{pmatrix}$. Show that PQ and RS are parallel.

Hint Find a number that multiplies one vector to give the other.

B4 $\overrightarrow{KL} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}$ and $\overrightarrow{MN} = \begin{pmatrix} -6 \\ 15 \end{pmatrix}$. Show that KL and MN are parallel. [2 marks]

Each question asks for something different. Show your working.

C1 The grid shows the vectors **a**, **b** and **c**.



(a) Write **a** as a column vector.

[1 mark]

Answer

(b) Write **b** as a column vector.

[1 mark]

Answer

(c) Write **c** as a column vector.

[1 mark]

Answer

Total for C1: 3 marks

C2 $\mathbf{a} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ -11 \end{pmatrix}$. Work out the length of the vector $2\mathbf{a} - \mathbf{b}$.

[3 marks]

Answer

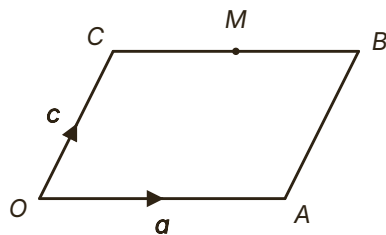
C3 $p \begin{pmatrix} 2 \\ 3 \end{pmatrix} + q \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 7 \end{pmatrix}$. Find the value of p and the value of q .

[3 marks]

Answer $p = \dots\dots$ $q = \dots\dots$

C4 A is the point $(1, 2)$, B is $(4, 8)$ and C is $(6, 12)$. Use vectors to show that A , B and C lie on a straight line. [3 marks]

C5 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of CB .



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(a) Find $\overrightarrow{AC}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2 marks]

Answer

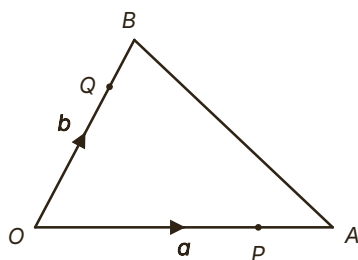
(b) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2 marks]

Answer

Total for C5: 4 marks

- C6** OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is the point on OA with $OP : PA = 3 : 1$, and Q is the point on OB with $OQ : QB = 3 : 1$. [3 marks]

Show that PQ is parallel to AB , and find the ratio $PQ : AB$.



Not drawn accurately

- C7** Each pair of vectors below is parallel.

- (a) $\begin{pmatrix} k \\ 6 \end{pmatrix}$ is parallel to $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Find k . [2 marks]

Answer

- (b) $\begin{pmatrix} -2 \\ m \end{pmatrix}$ is parallel to $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find m . [2 marks]

Answer

Total for C7: 4 marks

- C8** The point $A(-1, 3)$ is translated by $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ and then by $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$.

- (a) Write the single vector that does both translations. [1 mark]

Answer

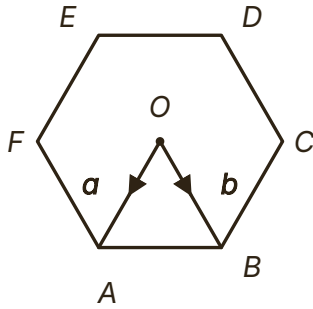
(b) Write down the coordinates of the final image of A .

[1 mark]

Answer

Total for C8: 2 marks

C9 $ABCDEF$ is a regular hexagon with centre O . $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[1 mark]

Answer

(b) Find $\overrightarrow{AD}$ in terms of $\mathbf{a}$.

[2 marks]

Answer

(c) Find $\overrightarrow{CE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[2 marks]

Answer

Total for C9: 5 marks

Answer every question in the space given. Show your working: most of the marks are for method.

- D1** Sam says, " $6\mathbf{a} - 2\mathbf{b}$ is parallel to $3\mathbf{a} - 2\mathbf{b}$, because $6\mathbf{a} - 2\mathbf{b}$ is 2 lots of $3\mathbf{a} - 2\mathbf{b}$." [2 marks]
 $\mathbf{a}$ and $\mathbf{b}$ are not parallel. Sam is wrong. Explain why.

Answer

- D2** $ABCD$ is a parallelogram. A is the point $(1, 2)$, B is $(6, 4)$ and D is $(8, -5)$. [3 marks]
Find the coordinates of C .

Answer

- D3** $\mathbf{a}$ and $\mathbf{b}$ are vectors that are not parallel.
 $\overrightarrow{OA} = 2\mathbf{a} + \mathbf{b}$, $\overrightarrow{OB} = \mathbf{a} + 3\mathbf{b}$, $\overrightarrow{OC} = k\mathbf{a} + 7\mathbf{b}$
The points A , B and C lie on a straight line.
(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1 mark]

Answer

- (b) Find the value of k . [3 marks]

Answer

(c) Find the ratio $AB : BC$.

[2 marks]

Answer

Total for D3: 6 marks

- D4** OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. P is a point on AB with $\overrightarrow{OP} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$. [3 marks]

Find the ratio $AP : PB$.

Answer

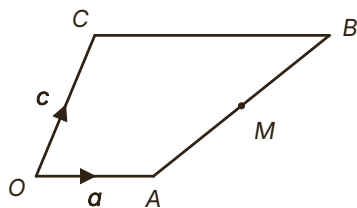
- D5** $\mathbf{p} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. The vector $\mathbf{p} + k\mathbf{q}$ is parallel to $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. [3 marks]

Find the value of k .

Answer

- D6** $OABC$ is a trapezium. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{CB} = 2\mathbf{a}$. M is the midpoint of AB . [3 marks]

Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. Simplify your answer.



Not drawn accurately

Answer

D7 $\overrightarrow{AB} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$. [3 marks]

Find the length of AC . Give your answer as a surd in its simplest form.

Answer

D8 The vector $\begin{pmatrix} k \\ k+7 \end{pmatrix}$ has length 13. [3 marks]

Find the two possible values of k .

Answer $k = \dots\dots$ or $k = \dots\dots$

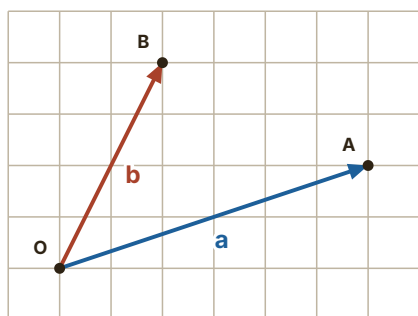
D9 $\mathbf{a}$ and $\mathbf{b}$ are vectors that are not parallel. [3 marks]

$$x\mathbf{a} + 3\mathbf{b} + 2(\mathbf{a} - y\mathbf{b}) = 7\mathbf{a} - 5\mathbf{b}$$

Find the value of x and the value of y .

Answer $x = \dots\dots$ $y = \dots\dots$

D10 The grid shows $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.



(a) Write $\mathbf{a} - \mathbf{b}$ as a column vector.

[2 marks]

Answer

(b) M is the midpoint of AB . Write $\overrightarrow{OM}$ as a column vector.

[2 marks]

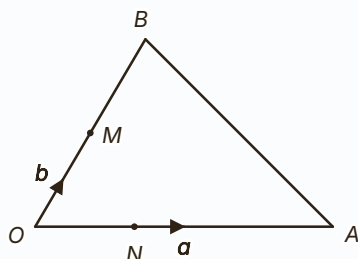
Answer

Total for D10: 4 marks

No route is given. Spend five minutes on a problem before you look at a hint. The hints are on the last page of the worksheet, and you can use one at a time.

- E1** OAB is a triangle. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is the midpoint of OB . N is the point on OA with $ON : NA = 1 : 2$. The lines AM and BN meet at X . [4 marks]

Find $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.



Not drawn accurately

Answer

- E2** $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. P is the point on OA with $OP : PA = 1 : 2$. The line CP meets the diagonal OB at X . [3 marks]

Find the ratio $OX : XB$.

Answer

E3 $ABCD$ is a trapezium with AB parallel to DC and $AB = 2DC$. The diagonals AC and BD meet at X . [3 marks]

Use vectors to show that $AX : XC = 2 : 1$.

E4 $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. The vector $\mathbf{b}$ is parallel to $\mathbf{a}$ but points the opposite way, and the length of $\mathbf{b}$ is 15. [3 marks]

Write $\mathbf{b}$ as a column vector.

Answer

F How did it go?

Colour one circle on each line.

	Not yet	Nearly	Yes
I can add, subtract and multiply column vectors, find the length of a vector, and read vectors off a grid.	☐	☐	☐
I can write a path round a shape in terms of given vectors, and use vectors to show that lines are parallel or that points lie on a straight line.	☐	☐	☐

Extension hints

Use one hint at a time. Cover the ones below it with a sheet of paper.

-
- E1** **Hint 1** Write $\overrightarrow{OX}$ two ways: once going along AM and once going along BN .
- Hint 2** $\overrightarrow{OX} = \mathbf{a} + s(\frac{1}{2}\mathbf{b} - \mathbf{a})$ and $\overrightarrow{OX} = \mathbf{b} + t(\frac{1}{3}\mathbf{a} - \mathbf{b})$.
- Hint 3** Match the $\mathbf{a}$ parts and the $\mathbf{b}$ parts: $1 - s = \frac{t}{3}$ and $\frac{s}{2} = 1 - t$. These give $s = \frac{4}{5}$.
-
- E2** **Hint 1** Write $\overrightarrow{OX}$ two ways: as a multiple of $\overrightarrow{OB}$, and by going from O to C and then part of the way along CP .
- Hint 2** $\overrightarrow{OX} = t(\mathbf{a} + \mathbf{c})$ and $\overrightarrow{OX} = \mathbf{c} + s(\frac{1}{3}\mathbf{a} - \mathbf{c})$.
- Hint 3** Match the $\mathbf{a}$ parts and the $\mathbf{c}$ parts: $t = \frac{s}{3}$ and $t = 1 - s$. These give $t = \frac{1}{4}$.
-
- E3** **Hint 1** Let $\overrightarrow{DC} = \mathbf{p}$ and $\overrightarrow{AD} = \mathbf{d}$. Then $\overrightarrow{AB} = 2\mathbf{p}$.
- Hint 2** $\overrightarrow{AX} = t(\mathbf{d} + \mathbf{p})$ along AC , and $\overrightarrow{AX} = 2\mathbf{p} + s(\mathbf{d} - 2\mathbf{p})$ along BD .
- Hint 3** Matching the parts gives $t = s$ and $t = 2 - 2s$, so $t = \frac{2}{3}$.
-
- E4** **Hint 1** Work out the length of $\mathbf{a}$ first.
- Hint 2** $\mathbf{a}$ has length 5, so $\mathbf{b}$ is 3 times as long.
- Hint 3** Opposite direction means a negative multiple: $\mathbf{b} = -3\mathbf{a}$.
-